

## ***Homework #2***

### ***Introduction to Communication Systems – E3701***

1 The definition of AM modulation is given by

$$x_{AM}(t) = A[1 + k_a s(t)] \cos(2\pi f_0 t).$$

Let  $x_1(t) = A[1 + k_a s_1(t)] \cos(2\pi f_0 t)$  and  $x_2(t) = A[1 + k_a s_2(t)] \cos(2\pi f_0 t)$ . If AM is

linear, we must have  $x_1(t) + x_2(t) = x_{1+2}(t)$ . But it is easy to verify

$$x_1(t) + x_2(t) \neq A[1 + k_a s_1(t) + k_a s_2(t)] \cos(2\pi f_0 t).$$

Hence AM modulation is not linear.

### Problem 2.3

The generation of an AM wave may be accomplished using various devices; here we describe one such device called a *switching modulator*. Details of this modulator are shown in Fig. P2.3a, where it is assumed that the carrier wave  $c(t)$  applied to the diode is large in amplitude, so that it swings right across the characteristic curve of the diode. We assume that the diode acts as an *ideal switch*, that is, it presents zero impedance when it is forward-biased [corresponding to  $c(t) > 0$ ]. We may thus approximate the transfer characteristic of the diode-load resistor combination by a *piecewise-linear characteristic*, as shown in Fig. P2.3b. Accordingly, for an input voltage  $v_1(t)$  consisting of the sum of the carrier and the message signal:

$$v_1(t) = A_c \cos(2\pi f_c t) + m(t) \quad (1)$$

where  $|m(t)| \ll A_c$ , the resulting load voltage  $v_2(t)$  is

$$v_2(t) \approx \begin{cases} v_1(t), & c(t) > 0 \\ 0, & c(t) < 0 \end{cases} \quad (2)$$

That is, the load voltage  $v_2(t)$  varies periodically between the values  $v_1(t)$  and zero at a rate equal to the carrier frequency  $f_c$ . In this way, by assuming a modulating wave that is weak compared with the carrier wave, we have effectively replaced the nonlinear behavior of the diode by an approximately equivalent piecewise-linear time-varying operation.

We may express Eq. (2) mathematically as

$$v_2(t) \approx A_c \cos(2\pi f_c t) + m(t)g_{T_0}(t) \quad (3)$$

where  $g_{T_0}(t)$  is a periodic pulse train of duty cycle equal to one-half, and period  $T_0 = 1/f_c$ , as in Fig. 1. Representing this  $g_{T_0}(t)$  by its Fourier series, we have

$$g_{T_0}(t) = \frac{1}{2} + \frac{2}{\pi} \sum_{n=1}^{\infty} \frac{(-1)^{n-1}}{2n-1} \cos[2\pi f_c t(2n-1)] \quad (4)$$

Therefore, substituting Eq. (4) in (3), we find that the load voltage  $v_2(t)$  consists of the sum of two components:

1. The component

$$\frac{A_c}{2} \left[ 1 + \frac{4}{\pi A_c} m(t) \right] \cos(2\pi f_c t)$$

which is the desired AM wave with amplitude sensitivity  $k_a = 4\pi A_c$ . The switching modulator is therefore made more sensitive by reducing the carrier amplitude  $A_c$ ; however, it must be maintained large enough to make the diode act like an ideal switch.

2. Unwanted components, the spectrum of which contains delta functions at  $0, \pm 2f_c, \pm 4f_c$ , and so on, and which occupy frequency intervals of width  $2W$  centered at  $0, \pm 3f_c, \pm 5f_c$ , and so on, where  $W$  is the message bandwidth.

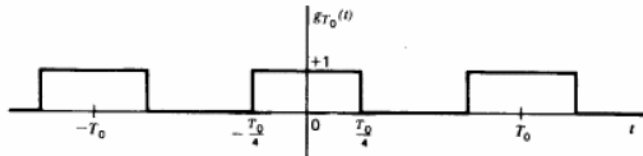


Fig. 1: Periodic pulse train

The unwanted terms are removed from the load voltage  $v_2(t)$  by means of a band-pass filter with mid-band frequency  $f_c$  and bandwidth  $2W$ , provided that  $f_c > 2W$ . This latter condition ensures that the frequency separations between the desired AM wave and the unwanted components are large enough for the band-pass filter to suppress the unwanted components.

### Problem 2.5

- (a) The demodulation of an AM wave can be accomplished using various devices; here, we describe a simple and yet highly effective device known as the *envelope detector*. Some version of this demodulator is used in almost all commercial AM radio receivers. For it to function properly, however, the AM wave has to be narrow-band, which requires that the carrier frequency be large compared to the message bandwidth. Moreover, the percentage modulation must be less than 100 percent.

An envelope detector of the series type is shown in Fig. P2.5, which consists of a diode and a resistor-capacitor (RC) filter. The operation of this envelope detector is as follows. On a positive half-cycle of the input signal, the diode is forward-biased and the capacitor  $C$  charges up rapidly to the peak value of the input signal. When the input signal falls below this value, the diode becomes reverse-biased and the capacitor  $C$  discharges slowly through the load resistor  $R_l$ . The discharging process continues until the next positive half-cycle. When the input signal becomes greater than the voltage across the capacitor, the diode conducts again and the process is repeated. We assume that the diode is ideal, presenting resistance  $r_f$  to current flow in the forward-biased region and infinite resistance in the reverse-biased region. We further assume that the AM wave applied to the envelope detector is supplied by a voltage source of internal impedance  $R_s$ . The charging time constant  $(r_f + R_s)C$  must be short compared with the carrier period  $1/f_c$ , that is

$$(r_f + R_s)C \ll \frac{1}{f_c} \quad (1)$$

so that the capacitor  $C$  charges rapidly and thereby follows the applied voltage up to the positive peak when the diode is conducting.

- (b) The discharging time constant  $R_l C$  must be long enough to ensure that the capacitor discharges slowly through the load resistor  $R_l$  between positive peaks of the carrier wave, but not so long that the capacitor voltage will not discharge at the maximum rate of change of the modulating wave, that is

$$\frac{1}{f_c} \ll R_l C \ll \frac{1}{W} \quad (2)$$

where  $W$  is the message bandwidth. The result is that the capacitor voltage or detector output is nearly the same as the envelope of the AM wave.

### Problem 2.6

Let  $v_1(t) = A_c(1 + k_S m(t)) \cos(2\pi f_c t)$

(a) Then the output of the square-law device is

$$v_2(t) = a_1 v_1(t) + a_2 v_1^2(t) = a_1 A_c(1 + k_S m(t)) \cos(2\pi f_c t) + \frac{1}{2} a_2 A_c^2 (1 + 2k_S m(t) + k_S^2 m^2(t) (1 + \cos(4\pi f_c t)))$$

(b) The desired signal,  $a_2 A_c^2 k_S m(t)$ , is due to the  $a_2 v_1^2(t)$ , hence the name “square-law detection”. This component can be extracted with a low-pass filter. This is not the only contribution within the baseband spectrum, because the term  $\frac{1}{2} a_2 A_c^2 k_S^2 m^2(t)$  will contribute with similar frequency components. The ratio of wanted signal to distortion is  $2/k_S m(t)$ . To make this ratio large, the percentage modulation, that is,  $|k_S m(t)|$  should be kept small compared with unity.

### Problem 2.9

The two AM modulator outputs are

$$s_1(t) = A_c(1 + k_S m(t)) \cos(2\pi f_c t)$$

$$s_2(t) = A_c(1 - k_S m(t)) \cos(2\pi f_c t)$$

Now  $s(t) = s_1(t) - s_2(t) = 2 k_S A_c m(t) \cos(2\pi f_c t)$ , which represents a DSB-SC modulated signal.

### Problem 2.10

(a) Multiplying the signal by the local oscillator gives:

$$s_1(t) = A_c m(t) \cos(2\pi f_c t) \cos(2\pi(f_c + \Delta f)t) = \frac{A_c}{2} m(t) [\cos(2\pi \Delta f t) + \cos(2\pi(2f_c + \Delta f)t)]$$

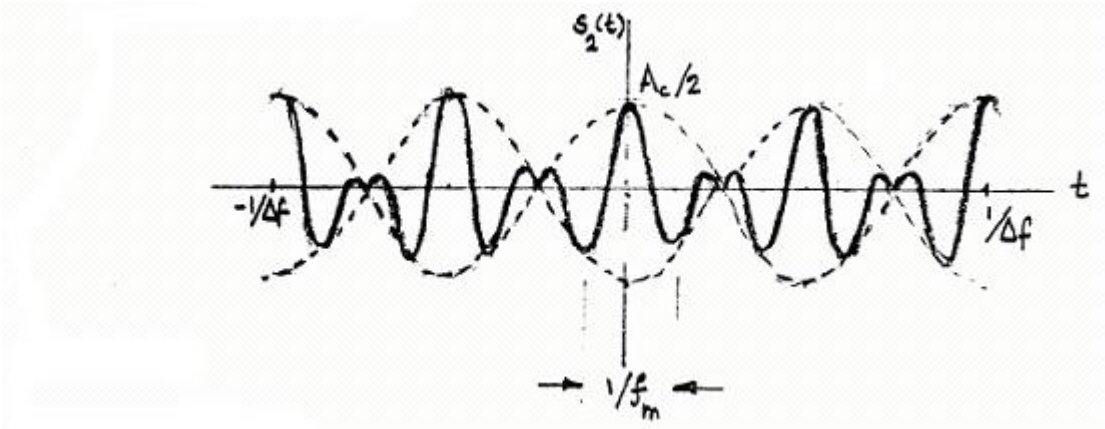
After a low-pass filter we have :

$$s_2(t) = \frac{A_c}{2} m(t) \cos(2\pi \Delta f t)$$

Thus the output signal is the message demodulated by a sinusoid of frequency  $\Delta f$

(b) If  $m(t) = \cos(2\pi f_m t)$ , then

$$s_2(t) = \frac{A_c}{2} \cos(2\pi f_m t) \cos(2\pi \Delta f t)$$



7 a) As shown in class, the input of a QAM demodulator is  $x_{QAM}(t) = s_1(t) \cos(2\pi f_0 t) - s_2(t) \sin(2\pi f_0 t)$ . At the demodulator, the output of the first branch is

$$\begin{aligned} & [s_1(t) \cos(2\pi f_0 t) - s_2(t) \sin(2\pi f_0 t)] 2 \cos(2\pi f_0 t + \theta) \\ &= 2s_1(t) \cos(2\pi f_0 t) \cos(2\pi f_0 t + \theta) - 2s_2(t) \sin(2\pi f_0 t) \cos(2\pi f_0 t + \theta) \\ &= s_1(t) \cos(4\pi f_0 t + \theta) + s_1(t) \cos(\theta) - s_2(t) \sin(4\pi f_0 t + \theta) + s_2(t) \sin(\theta) \end{aligned}$$

After a LPF, we have  $s_1(t) \cos(\theta) + s_2(t) \sin(\theta)$ .

Likewise, the output of the second branch is given by  $s_1(t) \sin(\theta) - s_2(t) \cos(\theta)$

b) When  $\theta = \frac{\pi}{2}$ , the output of the first branch is  $s_2(t)$  and the output of the second branch is

$$s_1(t)$$