Introduction to Communication Systems
Columbia University
ELEN E3701
Spring Semester- 2008

Problem Set #1

Problems Due: 5 February 2008

1. **The Fourier Transform, $\mathfrak{I}$ {s(t)}, is defined as S(f), and its inverse Fourier Transform, s(t), is defined as, $\mathfrak{I}^{-1}$ {S(f)},**
   See definitions below.
   We will use the Fourier Transform definition which replaces "ω" ("ω" is measured in radians/sec) with 2πf (where "f" is measured in cycles/sec or Hz).

$$S(f) = \mathfrak{I}\{s(t)\} = \int_{-\infty}^{\infty} s(t)\, e^{-j2\pi ft} dt$$

$$s(t) = \mathfrak{I}^{-1}\{S(f)\} = \int_{-\infty}^{\infty} S(f)\, e^{j2\pi ft} df$$

<u>Prove the following four properties of Fourier Transforms</u>

a. $\Im\{s(t-\tau)\}= S(f)\, e^{-j2\pi f\tau}$

b. $\Im\{s(t)\cos 2\pi f_0 t\}= S(f-f_0)/2+S(f+f_0)/2$

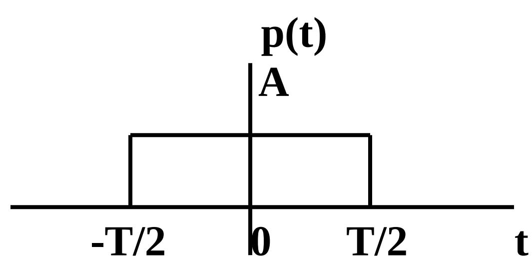
c. $S(0)=\int_{-\infty}^{\infty} s(t)\, dt$

d. $s(0)=\int_{-\infty}^{\infty} S(f)\, df$

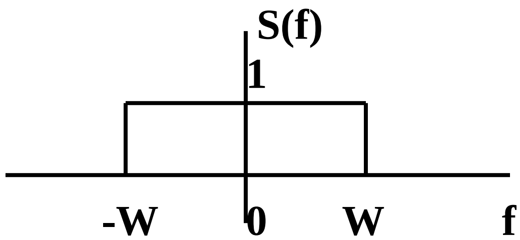
e. Show that the Fourier Transform, S(f), of an even function, s(t), is real and that the Fourier Transform, R(f), of an odd function, r(t), is imaginary.

f. Show that for the Fourier Transform, S(f), of a real function s(t), S(f) for negative frequencies is the complex conjugate of S(f), for the positive frequencies.

2. Find and draw the Fourier Transform of the rectangular pulse, p(t) shown below.



3. **Find and draw the Inverse Fourier Transform, s(t), of the ideal perfectly bandlimited rectangular spectrum, S(f), with bandwidth W.**



The results of Problems 2 and 3 above, illustrate the very important point that you cannot have a time signal, which is both perfectly time- limited and perfectly band –limited.
It also illustrates the duality principle of the Fourier Transform.

4. **Find and draw the spectrum of the signal, y(t),**

$$y(t)=s(t)\cos 2\pi f_0 t$$

if s(t) is the signal of Problem #3 above.

What is the bandwidth of the new signal, y(t), as compared to the bandwidth of the original, s(t).

5. **If Y(f)=H(f) X(f) prove that y(t) is the convolution of x(t) with h(t), i.e.,**

$$y(t)= h(t)*x(t)= \int_{-\infty}^{\infty} x(\tau)\, h(t-\tau)\, d\tau$$