

**Columbia University**

**Principles of Communication  
Systems  
ELEN E3701**

**Spring Semester- 2006**

**9 May 2006**

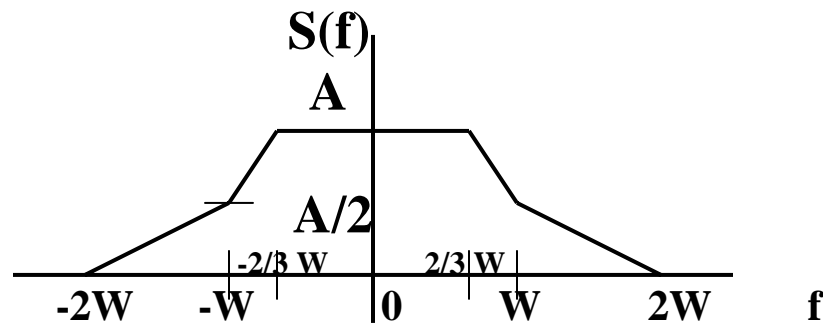
**Final Examination**

- **Length of Examination- 3 hours**
- **Answer All Questions**
- **Good Luck!!!**

**I. Kalet**

## Problem #1 (34 Points)

A pulse,  $s(t)$ , has the spectrum shown below



- a. Does this spectrum satisfy the Nyquist I Condition for NO ISI? Explain your answer.
- b. We are allowed to add on (or subtract) energy to this signal spectrum either in the frequency band
  - 1- Between  $W$  and  $2W$  Hz (and  $-W$  and  $-2W$  Hz),  
or in the band,
  - 2- Between  $-W$  to  $+W$  Hz, to create a new signal spectrum,  $H(f)$ .

**-Is it possible to meet the Nyquist I condition for NO ISI, by changing the spectrum only in the band between  $W$  and  $2W$  Hz (and  $-W$  and  $-2W$  Hz)? Explain your answer and show the new spectrum,  $H(f)$ , if it exists.**

**-Is it possible to meet the Nyquist I condition for NO ISI, by changing the spectrum only in the band between  $-W$  and  $W$  Hz? Explain your answer and show the new spectrum,  $H(f)$ , if it exists.**

**c. Which of your answers to part (b) has the smaller bandwidth and what is this bandwidth?**

**d. We now turn our attention to PCM systems**

**We would like to design one PCM communications system to handle the following three types of signals.**

**-Signal 1-Voice Signal with highest frequency = 1 KHz and maximum amplitude = 1 volt**

**-Signal 2-Voice Signal with highest frequency = 5 KHz and maximum amplitude = 10 volts**

**-Signal 3-Voice Signal with highest frequency = 10 KHz and maximum amplitude = 100 volts.**

**We will assume that each signal can be represented by a sine wave at the highest frequency.**

We would like the Signal-to-Quantization Noise ratio to be greater than 20 dB for each signal.

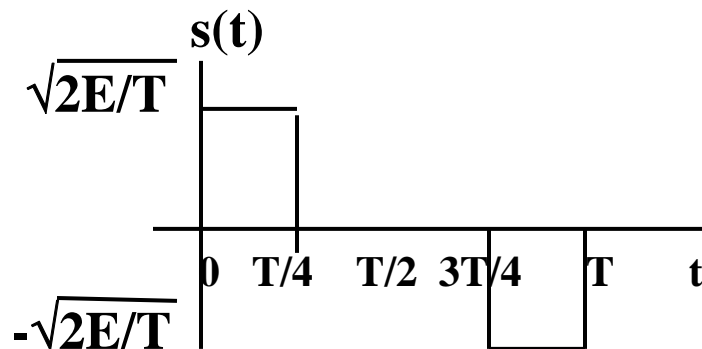
The PCM system, which we have designed has a sampler operating at 12 kilosamples per second, has a dynamic range from plus 100 volts to minus 100 volts and uses an eight bit quantizer to cover the entire dynamic range.

**-Is this PCM system good enough to handle all three signals?  
EXPLAIN YOUR ANSWER POINTING OUT WHICH SIGNALS MEET THE REQUIREMENTS AND WHICH DO NOT MEET THE REQUIREMENTS.**

## Problem #2 (33 points)

Parts a,b,c and d of this problem concern matched filtering.

We have the following pulse,  $s(t)$ , shown below.



- What is the energy,  $E$ , in this pulse?
- Find the matched filter,  $h_{MF}(t)$ , of this pulse and draw the impulse response,  $h_{MF}(t)$ , of this filter, if we want to sample exactly at  $t=T$  seconds.
- Draw the output of the matched filter, if  $s(t)$  is the input signal. Show all important points.

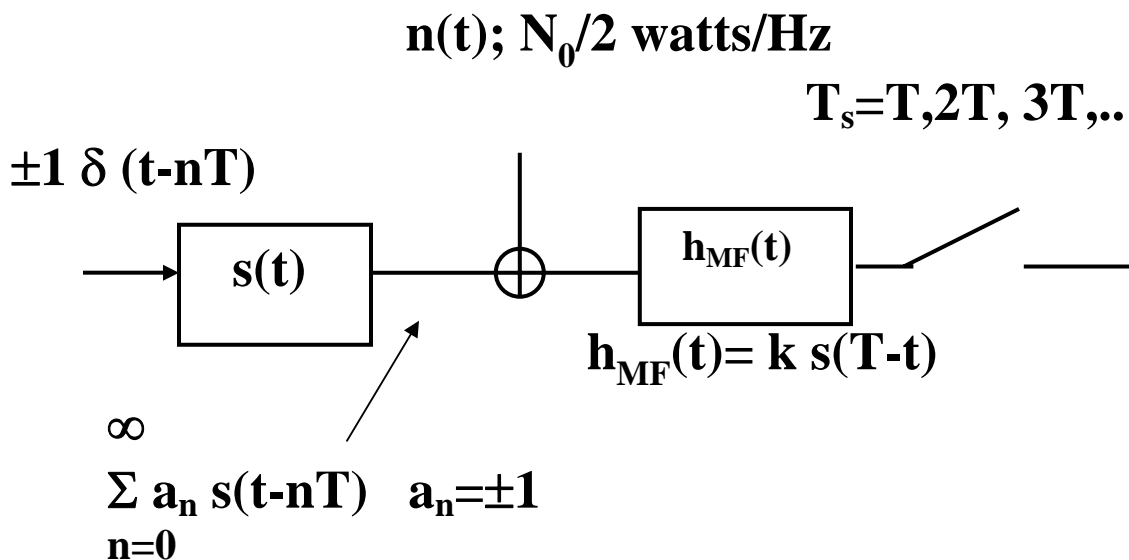
d. We now use the pulse  $s(t)$  as the baseband modulating pulse in the simple binary pulse amplitude modulation scheme, shown below.

We use the matched filter,  $h_{MF}(t)$ , as the receiver filter.

We sample every  $T$  seconds, exactly at the correct time, so that there is no ISI, and we make our decisions, based on these samples, and a threshold level equal to zero volts.

What is the probability of error of this binary scheme, in terms of  $E$  and  $N_0$ ?

### Pulse Amplitude Modulation System



**Part (e) below concerns binary modulations using optimum decision making at the receiver.**

- e. In some BPSK systems, a small amount of power is sent on a SINE carrier to ease the phase synchronization problem at the receiver.

In these systems, the two signals which are transmitted are either

$$x_1(t) = k \sqrt{2E_b/T} \cos 2\pi f_0 t + \sqrt{(1-k^2)} [-\sqrt{2E_b/T} \sin 2\pi f_0 t]$$

or

$$x_2(t) = -k \sqrt{2E_b/T} \cos 2\pi f_0 t + \sqrt{(1-k^2)} [-\sqrt{2E_b/T} \sin 2\pi f_0 t]$$

for  $0 \leq t \leq T$

$$0 \leq k \leq 1$$

**1-Draw a signal space representation of the two signals.**

2-What will be the probability of bit error as a function of  $E_b/N_0$ , and  $k$ , for this system, if we use the optimum decision making criterion at the receiver?

3-What is the probability of error for  $k=1$  and what is the probability of error for  $k=0$ ?

### **Problem #3 (33 points)**

We would like to decide which of two possible 64-ary systems to use, to transmit six bits per symbol, over an additive white gaussian noise channel. Basically, this means finding which of the signal constellations has the larger minimum distance between signals for the same average energy per symbol or bit (because there are six bits in each symbol for both modulations)

Here are the two candidates.

#### **System 1-**

In this system we transmit one of 16 QPSK signals, every symbol time. Each of the QPSK signals is on a different frequency ( $f_0+n/T$ ),  $n=0,1,2, \dots,15$ .

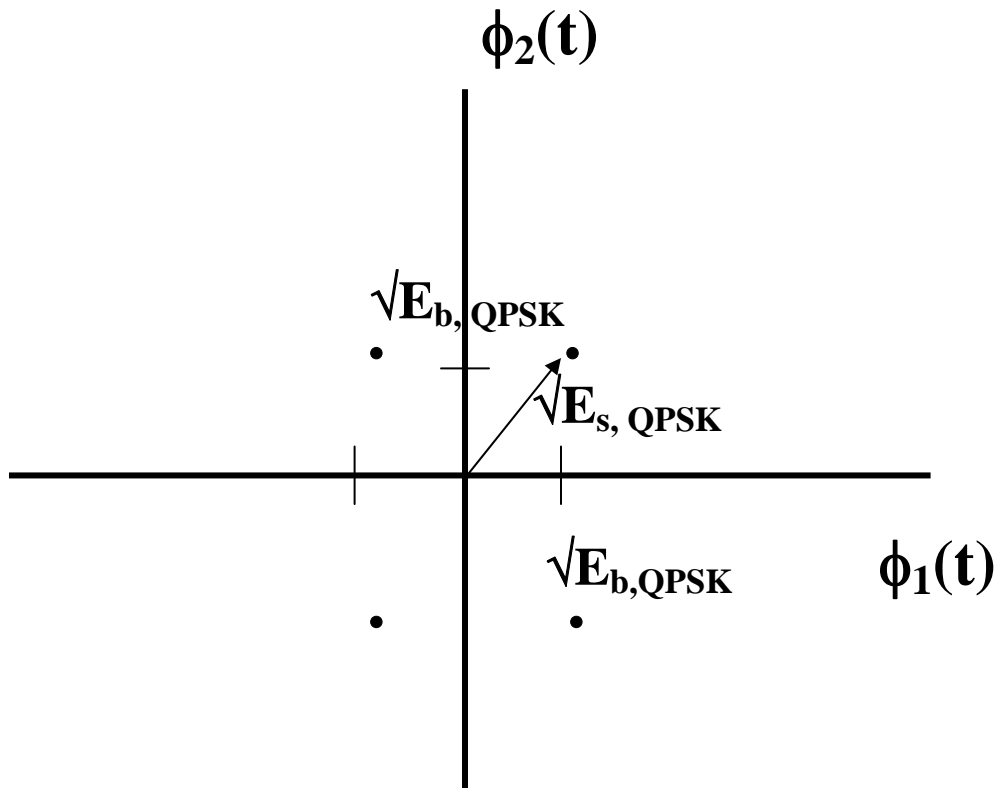
Each of the individual QPSK signals is described below

$$\mathbf{x}(t) = \mathbf{a}_1 \left[ \sqrt{2/T} \cos 2\pi(f_0+n/T)t \right] + \mathbf{a}_2 \left[ -\sqrt{2/T} \sin 2\pi(f_0+n/T)t \right] \quad ; 0 \leq t \leq T$$

$$\mathbf{a}_1, \mathbf{a}_2 = \pm \sqrt{E_{b,16\text{-QPSK}}} \cdot n=0,1,2, \dots,15$$

Every T seconds, we transmit one of these 64 possible signals.

## A Typical QPSK Constellation



$$\phi_1(t) = \sqrt{2/T} \cos 2\pi(f_0 + (n/T)) t$$

$$\phi_2(t) = -\sqrt{2/T} \sin 2\pi(f_0 + (n/T)) t$$

$$n=0,1,2, \dots,15$$

**System 2-**

**In this system we transmit one of 64 Orthogonal FSK signals.**

**Each signal may be described by the equation below**

$$x_i(t) = \sqrt{2E_{s, \text{FSK}}/T} \cos 2\pi [(f_0 + (n/T))] t ;$$

$$i = 0, 1, 2, 3, \dots, 63$$

- a. How many dimensions are there in each of the two proposed modulations?
- b. What is the minimum distance of the Orthogonal MFSK signal set, as a function of  $E_{\text{FSK}}$ ?
- c. What is the minimum distance of the 16-QPSK signal set, as a function of  $E_{s, \text{QPSK}}$ ?
- d. Which of the signal sets has a smaller minimum distance, if  $E_s$  is the same in each set? Explain.
- e. What is the maximum distance, between points in the Orthogonal 64-FSK signal set, as a function of “d”?
- f. What is the maximum distance, between points in the 16-QPSK signal set, as a function of “d”?

**End of Examination!!!!**

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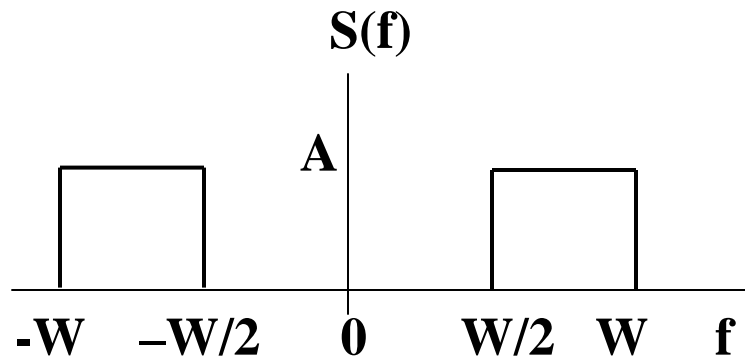
**Final Examination**

- **Length of Examination- Two hours and fifty minutes**
- **Answer All Questions**
- **Good Luck and Have a Nice Summer!!!**

**I. Kalet**

## Problem #1 (34 Points)

A pulse,  $s(t)$ , has the spectrum,  $S(f)$ , shown below



- a. Does this spectrum satisfy the Nyquist I Condition for NO ISI? Explain your answer.
  
- b. We are allowed to add on energy to this signal spectrum only in the frequency bands between  $W$  and  $2W$  Hz (and  $-W$  and  $-2W$  Hz).  
 Is it possible to meet the Nyquist I condition for NO ISI, by changing the spectrum only in the band between  $W$  and  $2W$  Hz (and  $-W$  and  $-2W$  Hz)? Explain your answer and show the new spectrum,  $H(f)$ , if it exists.

**In parts (c ) and (d ) we turn our attention to PCM systems**

**c. A student would like to design one PCM communications system to satisfy the signal-to-quantization noise ratio,  $S/ N_q$ , requirements, and the sampling frequency requirements for the following three types of signals,**

**-Signal 1-Voice Signal with highest frequency = 4 KHz and maximum amplitude = 10 volt and signal-to-quantization noise ratio,  $S/ N_q$ , > 40 dB.**

**-Signal 2-Music Signal with highest frequency = 15 KHz and maximum amplitude = 10 volts and signal-to-quantization noise ratio,  $S/ N_q$ , > 75 dB.**

**-Signal 3-A Video Camera Signal with highest frequency = 100 KHz and maximum amplitude = 100 volts and signal-to-quantization noise ratio,  $S/ N_q$ , > 82 dB.**

**We will assume that each signal can be represented by a sine wave at the highest frequency.**

**Our student thinks that if he (or she) designs her (or his) PCM system to satisfy the requirements of Signal 3, this design will be good enough for all the signals**

**What would be the minimum requirements on the student's design to satisfy the requirements of Signal 3?**

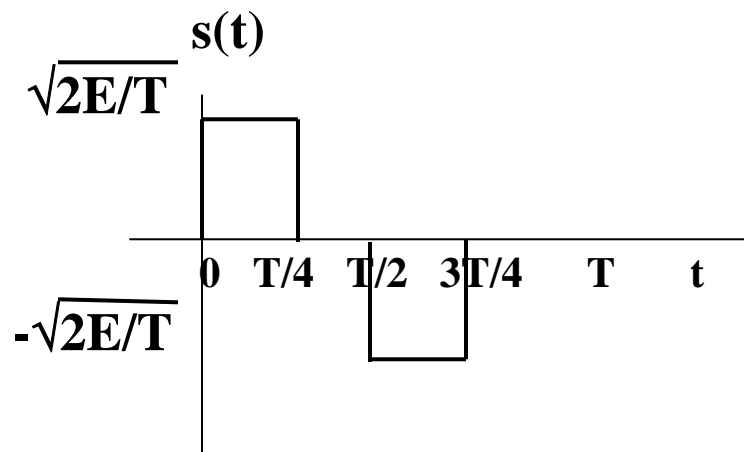
- 1-Minimum sampling frequency**
- 2-Minimum number of bits /sample**
- 3-Total bit rate in bps**

**d. Does the student's design satisfy the requirements for the other two signals? EXPLAIN YOUR ANSWER!!**

## Problem #2 (33 points)

This problem concerns matched filtering.

We have the following pulse,  $s(t)$ , shown below.

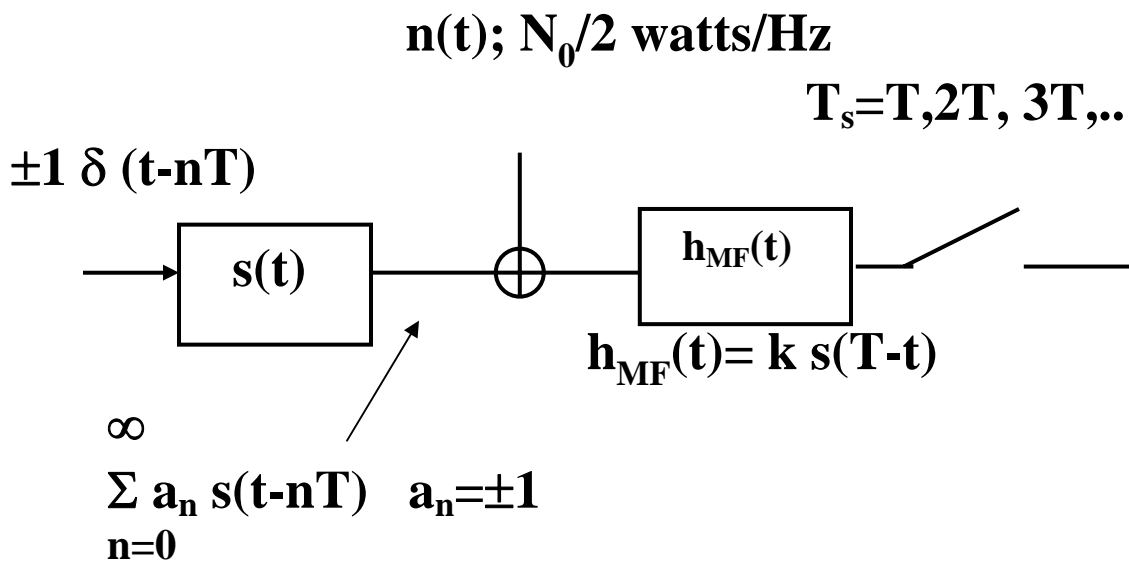


- What is the energy in this pulse?
- Draw the matched filter,  $h_{MF}(t)$ , of this pulse if we want to sample at time “ $t$ ” equal to “ $T$ ” seconds.
- Draw the output of the matched filter, if  $s(t)$  is the input signal. Show all important points.

d. We now use the pulse  $s(t)$  as the baseband modulating pulse in the simple binary pulse amplitude modulation scheme, shown below.

We use the matched filter,  $h_{MF}(t)$ , as the receiver filter. We sample every  $T$  seconds, exactly at the correct time, so that there is no ISI, and we make our decisions, based on these samples, and a threshold level equal to zero volts. What is the probability of error of this binary scheme, in terms of  $E$  and  $N_0$ ?

### Binary Pulse Amplitude Modulation System



### **Problem #3 (33 points)**

We would like to decide which of two possible 256-ary systems to use, to transmit eight bits per symbol, over an additive white gaussian noise channel. Basically, this means finding which of the signal constellations has the larger minimum distance between signals for the same average energy per symbol or bit (because there are six bits in each symbol for both modulations)

Here are the two candidates.

#### **System A-**

In this system we transmit one signal from one of sixteen 16-QAM signals, every symbol time. Each, of the 16-QAM signal sets, is on a different frequency ( $f_0+n/T$ ),  $n=0,1,2, \dots,15$ .

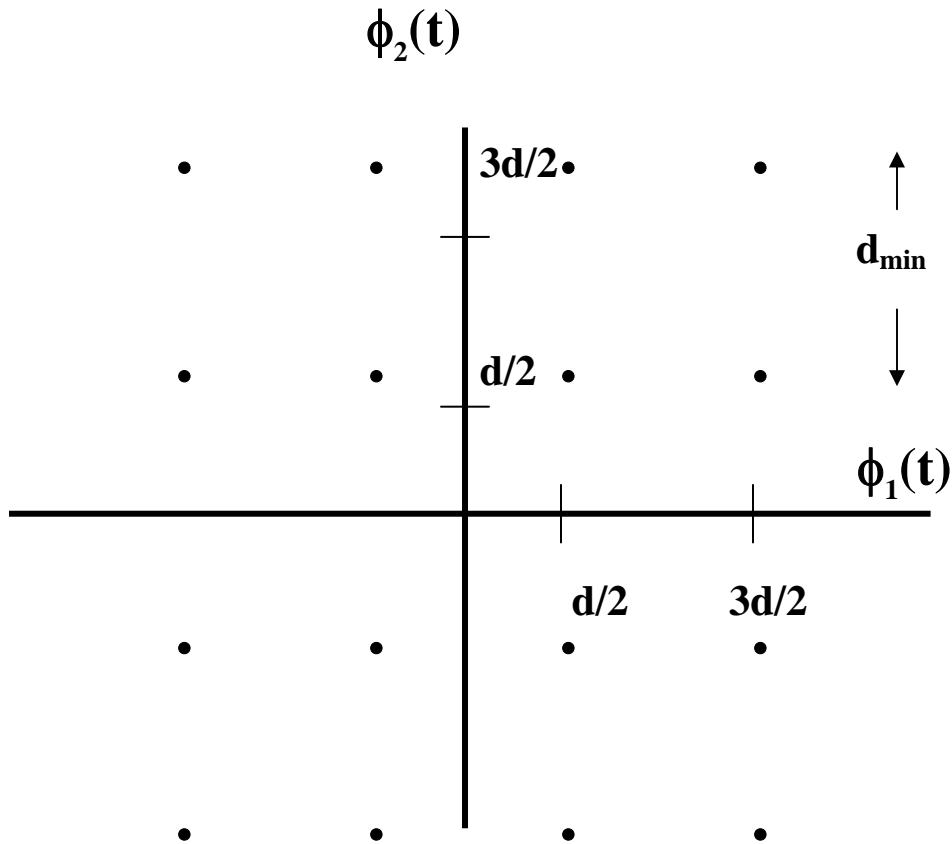
Each of the individual 16-QAM signal sets is described below

$$\mathbf{x}(t) = \mathbf{a}_1 \left[ \sqrt{2/T} \cos 2\pi(f_0+n/T)t \right] + \mathbf{a}_2 \left[ -\sqrt{2/T} \sin 2\pi(f_0+n/T)t \right] \quad ; 0 \leq t \leq T$$

$$\mathbf{a}_1, \mathbf{a}_2 = \pm d/2, \pm 3d/2 \\ n=0,1,2, \dots,15$$

Every T seconds, we transmit one of these 256 possible signals.

## A Typical 16-QAM Constellation



$$\phi_1(t) = \sqrt{2/T} \cos 2\pi(f_0 + (n/T)) t$$

$$\phi_2(t) = -\sqrt{2/T} \sin 2\pi(f_0 + (n/T)) t$$

$$n=0,1,2, \dots,15$$

## System B

In this system we transmit one of 256 Orthogonal FSK signals.

Each signal may be described by the equation below

$$x_i(t) = \sqrt{2E_{s, \text{FSK}}/T} \cos 2\pi [(f_0 + (n/T)) t];$$

$$n=0,1,2,3,\dots,255$$

where  $E_s$  is the average energy per symbol.

- How many dimensions are there in each of the two proposed modulations?
- What is the minimum distance of the Orthogonal MFSK signal set, System B, as a function of  $E_{\text{FSK}}$ ?  
How many signals are at this minimum distance for a given transmitted signal?
- What is the average energy,  $E_{s,\text{avg-QAM}}$  of the first modulation technique, System A, in terms of  $d$ ?
- What is the minimum distance of the System A modulation technique in terms of  $d$ ?
- Which of the signal sets has a bigger minimum distance, if  $E_s$  is the same in each set? (This is the modulation technique which you would like to use!)
- What is the maximum distance, between points in the **Orthogonal 256-FSK** signal set, as a function of " $E_s$ "?
- What is the maximum distance, between points in the System B signal set, as a function of " $E_s$ " or " $d$ "?

**End of Examination!!!!**