Principles of Communication Systems
Columbia University
ELEN E3701
Spring Semester- 2006

9 March 2006
Professor I. Kalet

Midterm Examination
(with corrections after exam)

- **Length of Examination- 1:15 hours**
- **Answer Both questions**
- **Each question is worth 50 points**

Good Luck and
Have a Nice Vacation!!

Problem #1 (50 Points)

The following FM signal is transmitted.

$$x_{FM}(t) = A \cos \{2\pi f_0 t + 2\pi h \int_{-\infty}^{t} s(\tau)\, d\tau\}$$

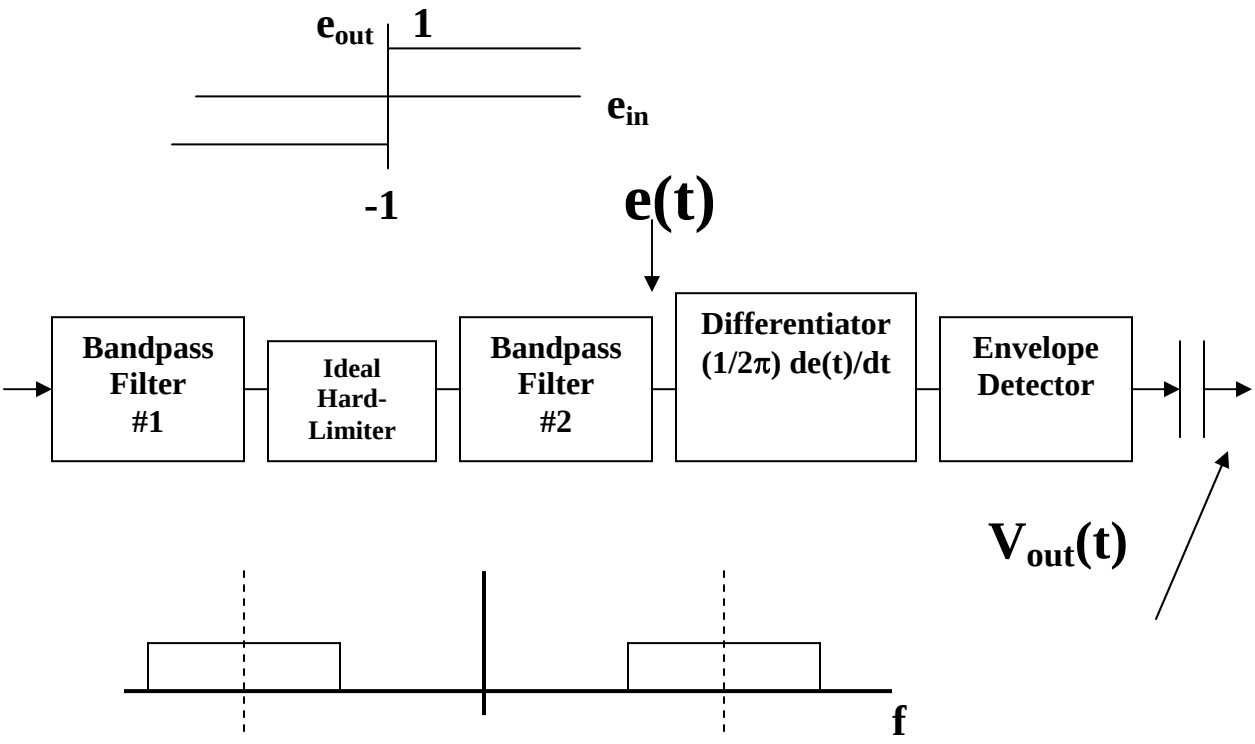
where s(t) is equal to $A_m \cos 2\pi W t$.

a. **Write down the equation for the $x_{FM}(t)$ after integrating s(t).**
b. **What is the maximum frequency deviation, Δf, in terms of h and A_m?**
c. **We now use an FM detector, as described in class, and shown on the next page.**

What is the output signal, $v_{out}(t)$ of the FM detector?

(Parts a, b and c are worth a total of 10 points all together)

FM Detector



Carson Bandwidth = 2 Δf + 2W

The <u>filter bandwidths of both Filters 1 and 2</u> are equal to the <u>Carson bandwidth of the FM signal.</u>

d. (20 points)

Suppose that we are still using the same FM detector shown above, but the input signal is now an AM signal, $x_{AM}(t)$ given by the equation below

$$x_{AM}(t) = A[1+ms(t)] \cos 2\pi f_0 t$$

where $0 < m < 1$

and

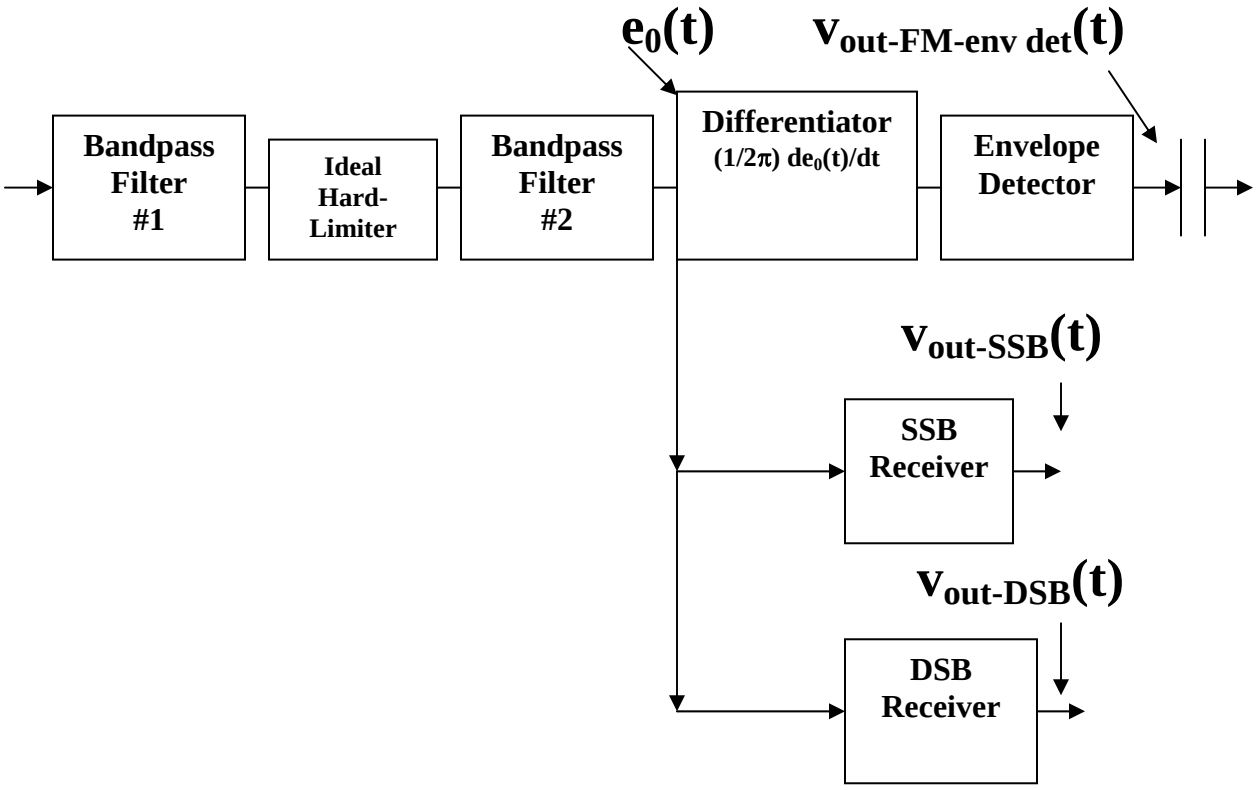
$$s(t) = B \cos 2\pi W t.$$

Find the output signal of the FM receiver if $x_{AM}(t)$ is the input signal.

where
$f_0 >>>> W$

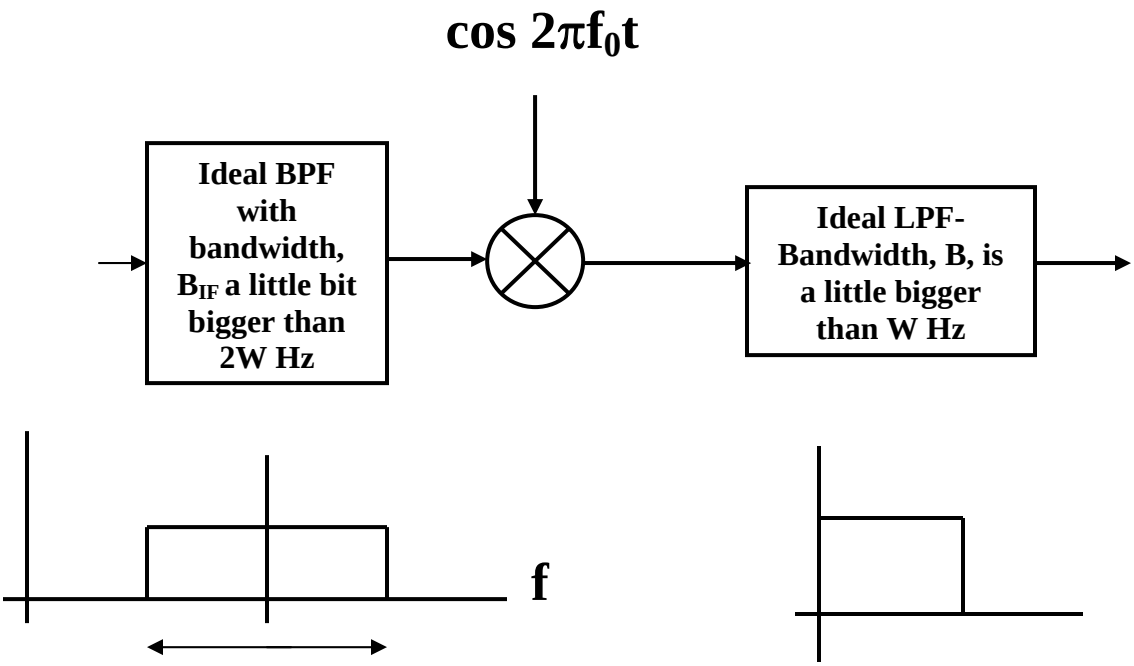
e. (20 points)

Suppose we now add on the following two branches, shown below, to our FM receiver.

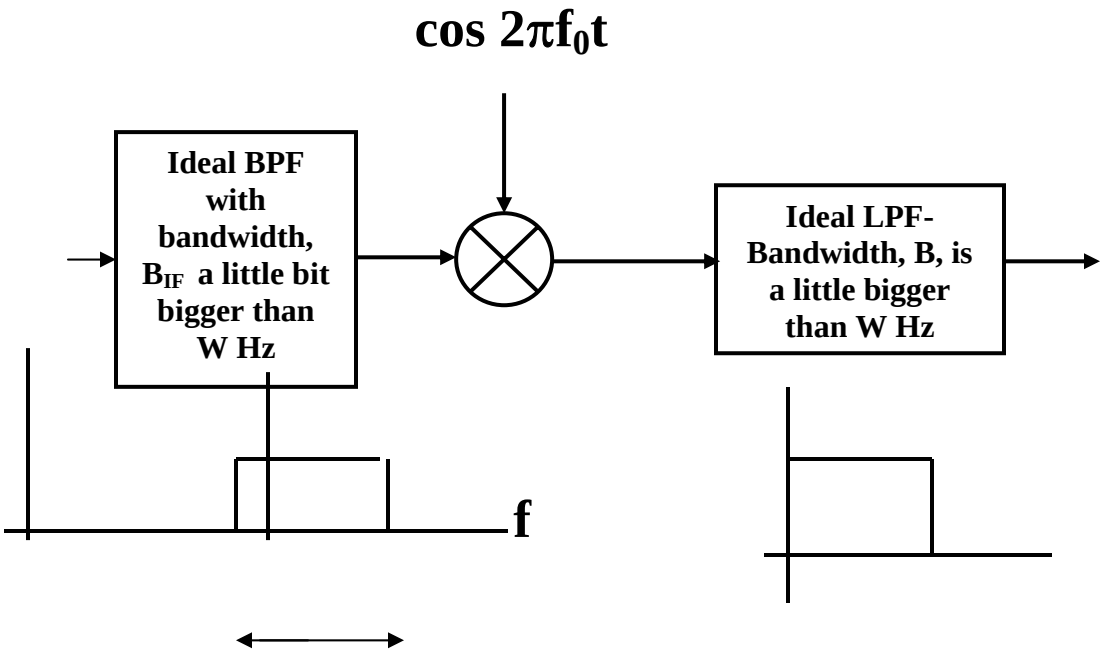


What will be the outputs $v_{out\text{-}SSB}(t)$ and $v_{out\text{-}DSB}(t)$, if the input signals are the FM (<u>now with β=2.2</u>) and AM signals of the previous parts.

DSB Receiver



SSB Receiver



Problem #2 (50 Points)

We have the following single-sideband signal

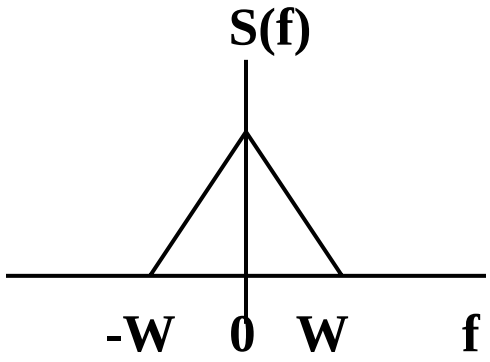
$$x_{USB\text{-}SSB}(t) = s(t)\cos 2\pi f_0 t - \hat{s}(t)\sin 2\pi f_0 t$$

We now use this signal to single-sideband, (upper-sideband) modulate the carrier , f_D

$$f_D >> f_0$$

$$f_0 >> W.$$

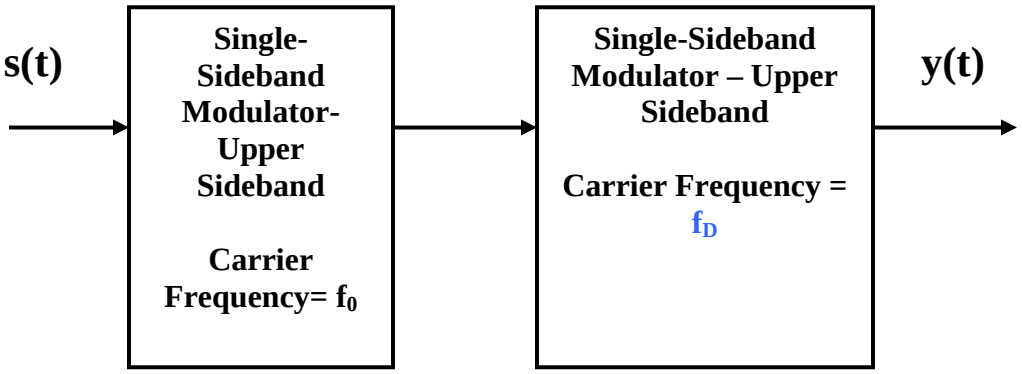
The function, s(t), has the spectrum shown below.



a. (35 points) Write the analytic expression for the resulting signal, y(t), and draw its spectrum.

b. (15 points) Build a receiver to recover the signal, s(t).

The combined DSB –SSB (USB) Modulator



The End!!!

sin x sin y= ½ [cos (x-y) - cos (x+y)]

cos x cos y= ½ [cos (x-y)+cos (x+y)]