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HOW TO SIMULATE MIXER NOISE?

Case A: $50\ \Omega$ *source with a resistive load*

Consider a very simple mixer as shown in the figure below. There is an RF source at 2.45 GHz with an amplitude of 100 mV. The switch is ideal and is driven by a sinusoidal LO of frequency 2.5 GHz. The source impedance is $50\ \Omega$ and the load is R_{load}. For simpler calculation, the load resistance is modeled as a noiseless *voltage controlled current source*.

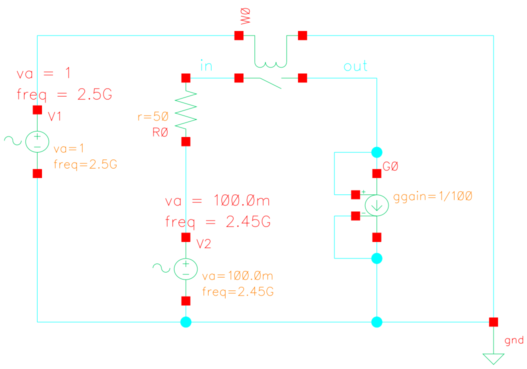


Fig. 1. Setup

First find the small signal voltage conversion gain (CVG) from RF input to the IF output.

$$
\begin{aligned}
V_{out} &= \frac{R_l}{R_s + R_l} V_{RF}, \quad \text{When } V_{LO} \text{ is high} \\
V_{out} &= 0, \quad \text{When } V_{LO} \text{ is low, } V_{out} \text{ discharges through } R_2 \\
V_{out} &= \frac{R_l}{R_s + R_l} A_{RF}\cos(\omega_{RF}t)\mathrm{sq}_{1,0}(t) \\
V_{out} &= \frac{R_l}{R_s + R_l} A_{RF}\cos(\omega_{RF}t)\Big\{\frac{1}{2} + \frac{2}{\pi}\Big(\cos(\omega_{LO}t) - \frac{1}{3}\cos(3\omega_{LO}t) + ...\Big)\Big\}
\end{aligned}
$$

IF component,

$$
\begin{aligned}
V_{out} &= \frac{1}{\pi}\frac{R_l}{R_s + R_l} A_{RF}\cos((\omega_{LO} - \omega_{RF})t) \\
\text{So,} & \\
CVG &= \frac{1}{\pi}\frac{R_l}{R_s + R_l}
\end{aligned}
$$

If in place of V_{rf}, we had a **white noise** ($\overline{V_{in}^2} = 4kTR_s\Delta f$) source due to the resistance R_s then at the IF output we would get output noise $\overline{V_{on}^2}$ given by (see Appendix),

$$
\overline{V_{on}^2} = \frac{1}{4}\Big\{\frac{R_l}{R_s + R_l}\Big\}^2 4kTR_s\Delta f
$$

The noise calculated above is *Single Sideband*. This is so because the input RF signal is *single-sideband*, since we only considered the case where $\omega_{LO} - \omega_{RF} = \omega_{IF}$.

For an input level of 100 mV (-20 dB) and $R_s = 50\ \Omega$, simulation[1] gave:

R_l	CVG (calc.)	Output (siml.)	CVG (siml.)	IF-noise (calc.)	IF-noise (siml.)
1 kΩ	-10.37 dBV	-30.45 dBV	-10.45 dBV	-187.264 dB	-188.2 dB
100 Ω	-13.62 dBV	-33.62 dBV	-13.47 dBV	-190.36 dB	-191.38 dB

Fig. 2 shows simulation results for $R_l = 100\ \Omega$.

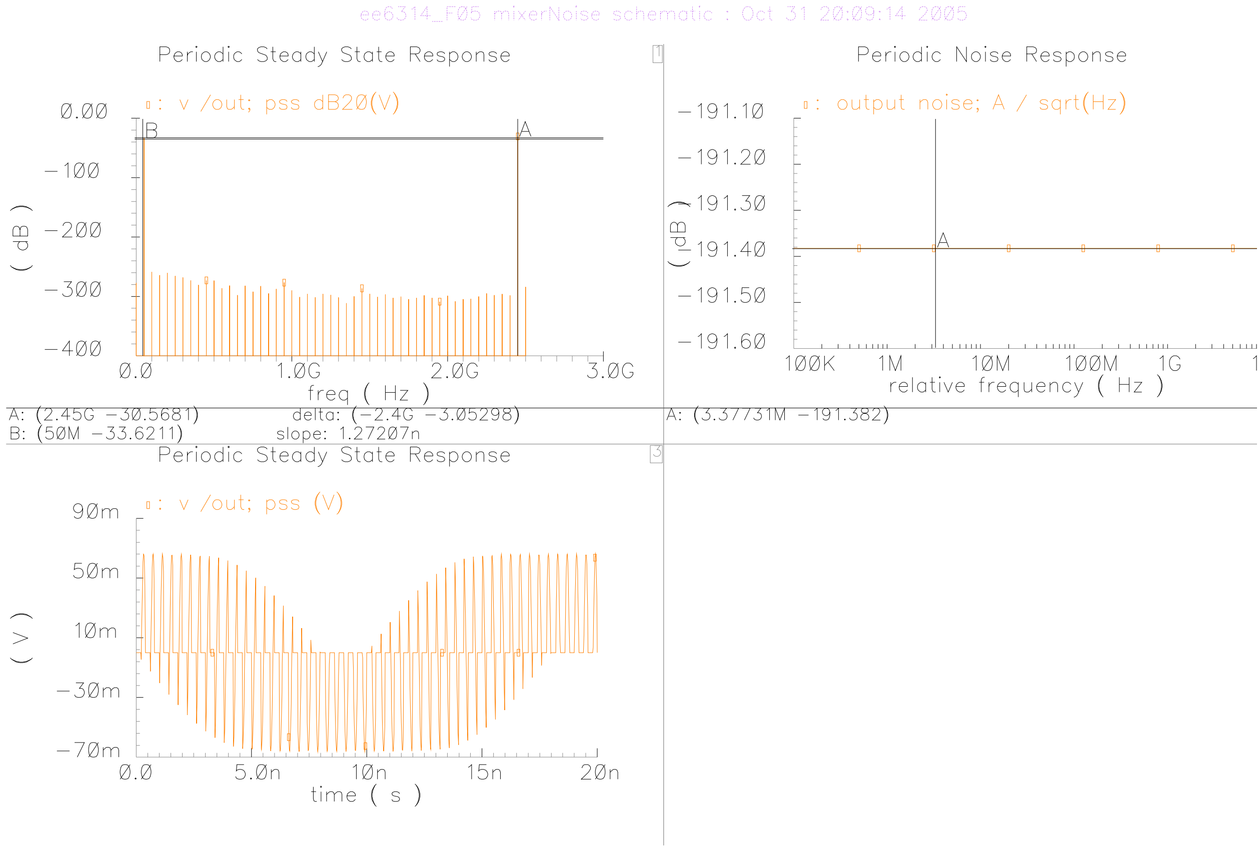


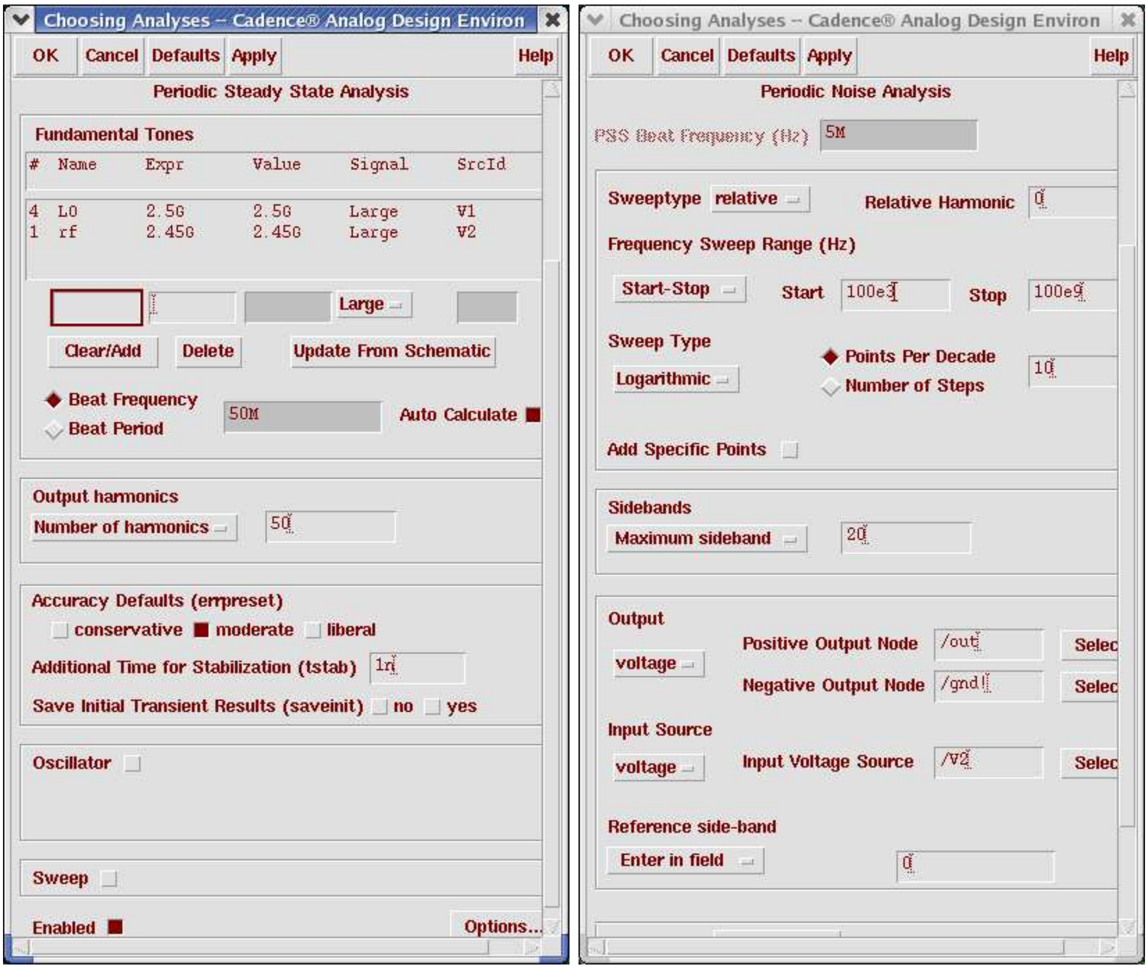
Fig. 2. Results for $R_l = 100\ \Omega$ case.

The entries in the *pss* and *pnoise* analysis form look like as shown below.

If the RF signal is sufficiently small then *pxf/pac* can be used to find the small signal transfer function/gain from RF to the output in the presence of LO.

Notice that in the pnoise setup, sweep type is relative. If the relative harmonic is x and the reference sideband is y. Then the noise is calculated for (100 kHz to 10 GHz) + $(x + y) \times 50$ MHz. If x is 0 and the reference sideband is 1, then relative sweep type would mean that the noise is calculated for (100 kHz to 10 GHz) + 1×50 MHz. Similarly for reference sideband $= -1$, the noise is simulated for (100 kHz to 10 GHz) + $(-1) \times 50$ MHz.

[1] All the simulations were done using TSMC018_teaching.scs model card.



(a) (b)

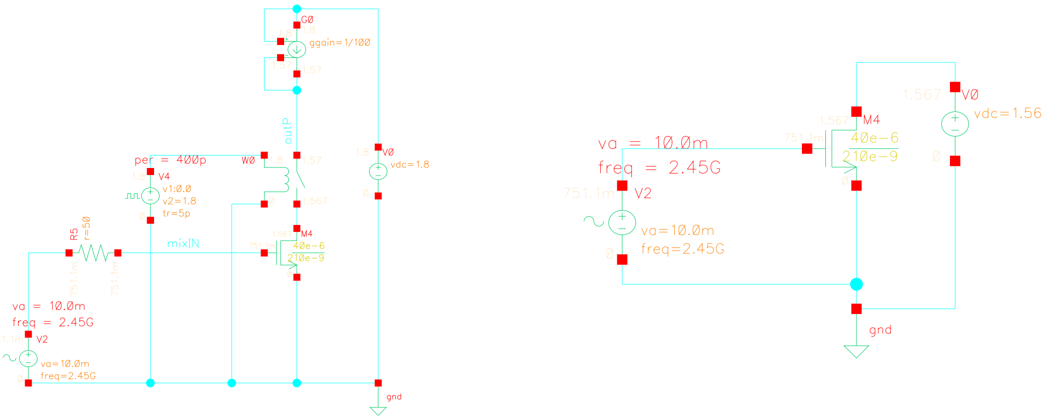
Fig. 3. (a) PSS Setup, (b) PNOISE setup

You can notice this in the *spectre.out* file when the simulation finishes. For the circuit presented above, you will not notice any change in the noise spectrum since the noise is white and the CVG is independent of the IF frequency. Try adding a capacitor in parallel with the load and then play with the *pnoise* settings to see the difference.

Case B: Noise figure of a Mixer with transconductor in the tail

Fig. 4(a) shows a mixer circuit using a transconductor stage at the RF input. LO drives an ideal switch. Noise figure is defined here for $50\ \Omega$ source impedance.

I first characterized the drain current noise power spectral density ($\overline{i_{dn}^2}$) of the nFET working as a transconductor in fig. 4(a). Fig. 4(b) shows the setup for noise characterization of the nFET (W/L = $40\ \mu$m/210 nm). Noise summary was read from *Analog Design Env.* $\rightarrow$ *Results* $\rightarrow$ *Print* $\rightarrow$ *Noise Summary* $\rightarrow$ *Spot Noise at* 50 *MHz (Include All Types)*. The noise summary also gives the input referred noise voltage PSD ($\overline{v_{gn}^2}$).



(a) (b)

Fig. 4. (a) Mixer with a transconductor stage, (b) Setup to characterize the noise of the nFET used as transconductor in the mixer.

At the input now we have two noise sources — $4kTR_s$ and $\overline{v_{gn}^2}$ which undergo same transformation because of mixing action. At the output we get $m \times (g_m R_l)^2$ times the input noise PSD. Same folding factor m as in Case A appears here. The major change is due to the noise of the transconductor. The folded noise PSD at the IF frequency of 50 MHz is, therefore, given by

$$\begin{aligned}
\overline{v_{on}^2} &= (4kTR_s + \overline{v_{gn}^2}) \times (g_m R_l)^2 \times m \\
&= (8.28 \times 10^{-19} + 4.4629 \times 10^{-18}) \times (13.5 \times 10^{-3} \times 100)^2 \times \frac{1}{4} \\
&= 2.4107 \times 10^{-18} \text{ V}^2/\text{Hz} \\
&= -176.18 \text{ dB}
\end{aligned}$$

Simulation result for the mixer circuit is presented in fig. 5. The RF signal (2.45 GHz) amplitude is 10 mV (-40 dB). LO frequency is 2.5 GHz. The output amplitude at 50 MHz is -47.73 dB.

The table below summarizes all the information.

W/L	V_{GS}	V_{DS}	g_m	$\overline{i_{dn}^2}$	$\overline{v_{gn}^2}$
40 μm/210 nm	751.1 mV	1.567 V	13.5 mS	8.1252×10^{-22} A^2/Hz	4.4629×10^{-18} V^2/Hz

$\overline{v_{on}^2}$ (calc.)	$\overline{v_{on}^2}$ (siml.)	CVG (calc.)	CVG (siml.)	NF
-176.18 dB	-176.53 dB	$\frac{g_m R_l}{\pi} = -7.34$ dBV	-7.73 dBV	11.37 dB

Case C: What happens in balanced LO case?

The circuit is shown in fig. 6. In this case, noise power from the two sides will add leading to 3 dB higher output noise power spectral density. Though the noise originate from the same source *viz.* the transconductor, due to the non-linear operation of the switches,

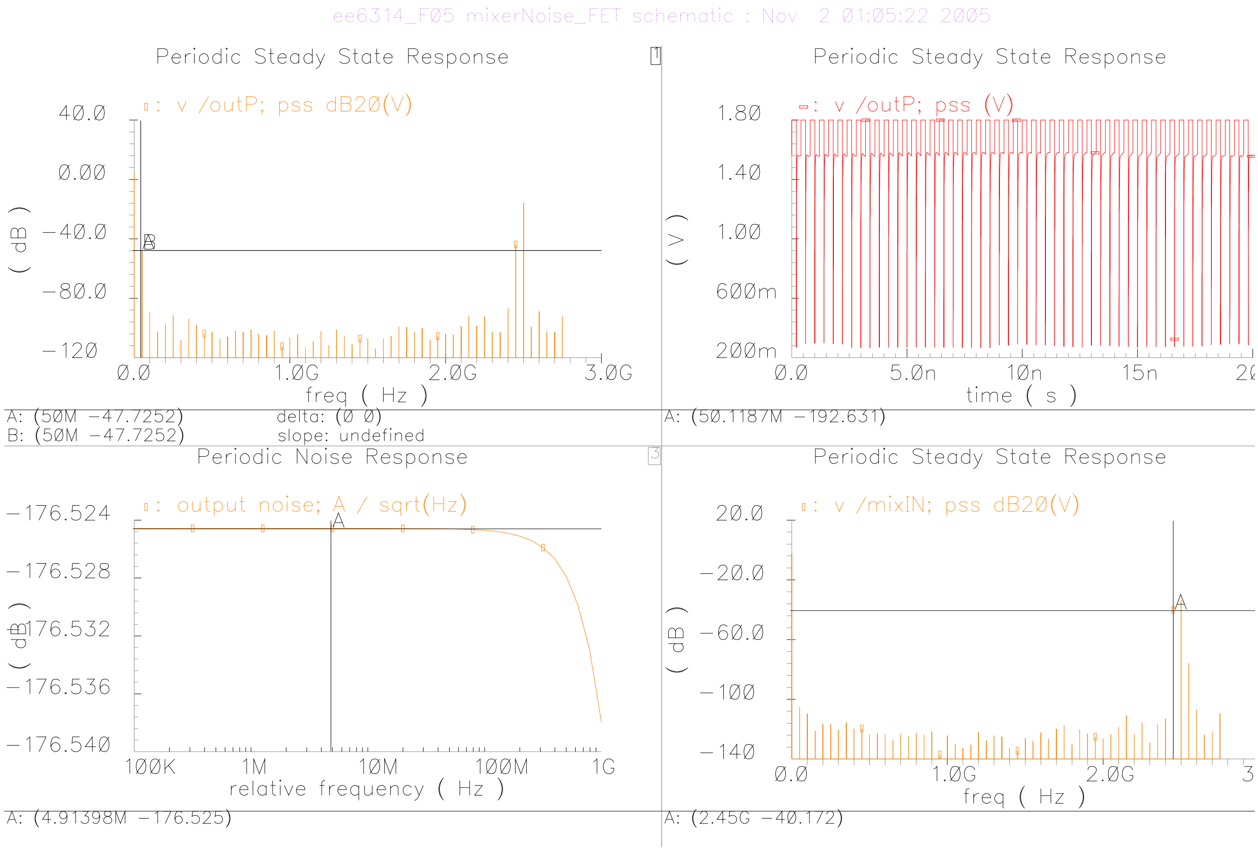


Fig. 5. Simulation results for mixer of fig. 4(a) (unbalanced RF, unbalanced LO)

the noise power add up instead of cancellation. The cancellation would have happened in a differential amplifier where the noise due to the tail current source do not appear in the differential output. Here, the total output noise power goes up by a factor of 2 (3 dB). The gain from RF source to the IF however doubles (in terms of CVG). NF will therefore decrease by a factor of 2 (3 dB) compared to unbal-LO, unbal-RF case.

$$\overline{v_{on}^2} = (4kTR_s + \overline{v_{gn}^2}) \times (g_m R_l)^2 \times m \quad \text{where m is 1/4}$$
$$CVG = \frac{2}{\pi} g_m R_l$$

APPENDIX

A. Derivation of noise folding factor m

For the circuit in fig. 1, the output noise appears due to folding of the $4kTR_s$ noise when multiplied with a $sq_{0,1}(t)$ function.

In the frequency domain, the white noise can be interpreted as consisting of impulses at each frequency. Consider the output noise contribution from components at ω_{IF}, $\omega_{LO} - \omega_{IF}$

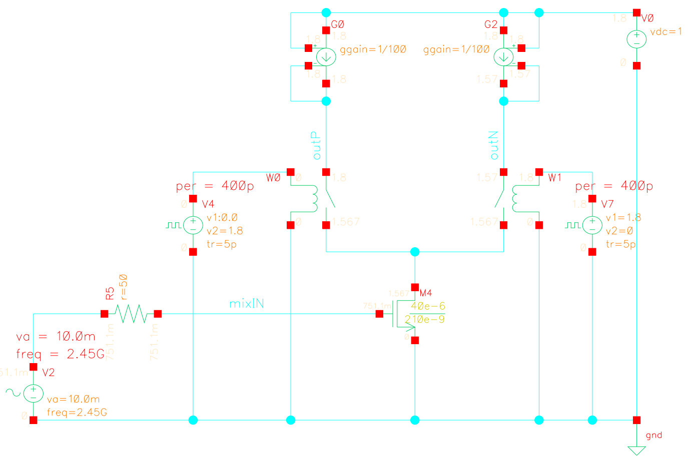


Fig. 6. Differential LO and unbalanced RF signal.

(or $\omega_{LO}+\omega_{IF}$), $3\omega_{LO}-\omega_{IF}$ (or $3\omega_{LO}+\omega_{IF}$)... at ω_{IF}.

$$\mathrm{sq}_{0,1}(t) \quad = \quad \frac{1}{2}+\frac{2}{\pi}\Big\{\cos(\omega t)-\frac{1}{3}\cos(3\,\omega t)+\frac{1}{5}\cos(5\omega t)...\Big\}$$

Each term in the above equation causes noise to appear at ω_{IF} as shown in fig. 8.

Term	Noise component	Folded component
$\frac{1}{2}$	ω_{IF}	$\frac{1}{2}\cos\omega_{IF}t$
$\frac{2}{\pi}\cos\omega_{LO}t$	$\omega_{LO}-\omega_{IF}$	$\frac{1}{\pi}\cos\omega_{IF}t$
$\frac{1}{3}\frac{2}{\pi}\cos 3\omega_{LO}t$	$3\omega_{LO}-\omega_{IF}$	$\frac{1}{3}\frac{1}{\pi}\cos\omega_{IF}t$
$\frac{1}{5}\frac{2}{\pi}\cos 5\omega_{LO}t$	$5\omega_{LO}-\omega_{IF}$	$\frac{1}{5}\frac{1}{\pi}\cos\omega_{IF}t$
...	...	...

The noise add in power since each term is uncorrelated. Noting that the average power of $\cos(\omega_{IF}t)$ is $1/2$, the output noise PSD is $m\times 4kTR_s$, where m is given by

$$\begin{aligned} m &= \frac{1}{2}\frac{1}{2^2}+\frac{1}{2}\Big(\frac{1}{\pi}\Big)^2+\frac{1}{2}\Big(\frac{1}{\pi}\Big)^2\frac{1}{3^2}+\frac{1}{2}\Big(\frac{1}{\pi}\Big)^2\frac{1}{5^2}+... \\ &= \frac{1}{8}+\frac{1}{2\pi^2}\times\Big(1+\frac{1}{3^2}+\frac{1}{5^2}+...\Big) \\ &= \frac{1}{8}+\frac{1}{2\pi^2}\frac{\pi^2}{8}=\frac{3}{16} \end{aligned}$$

Now taking into account the noise from both sides of LO, then

$$m \quad = \quad \frac{1}{8}+2\times\{\frac{1}{2\pi^2}\frac{\pi^2}{8}\}=\frac{1}{4}$$

If we had $\mathrm{sq}_{-1,1}(t)$, and we considered noise from just one side of LO, then

$$\begin{aligned} m &= \frac{1}{2}\Big(\frac{2}{\pi}\Big)^2+\frac{1}{2}\Big(\frac{2}{\pi}\Big)^2\frac{1}{3^2}+\frac{1}{2}\Big(\frac{2}{\pi}\Big)^2\frac{1}{5^2}+... \\ &= \frac{2}{\pi^2}\times\frac{\pi^2}{8}=\frac{1}{4} \end{aligned}$$

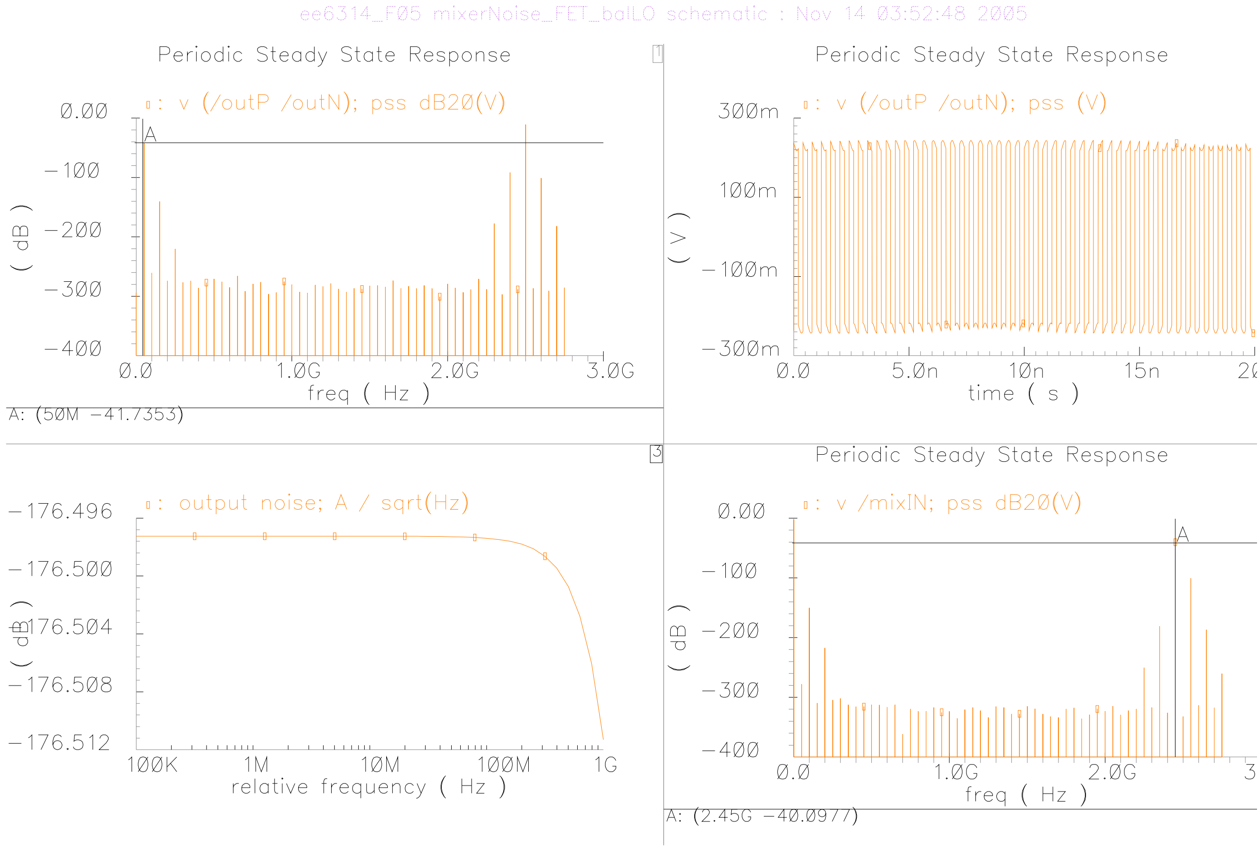


Fig. 7. Simulation results for mixer of fig. 6 (unbalanced RF, balanced LO)

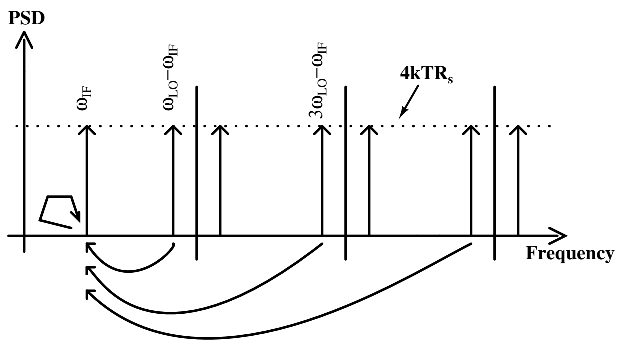


Fig. 8. Noise Folding in a Mixer

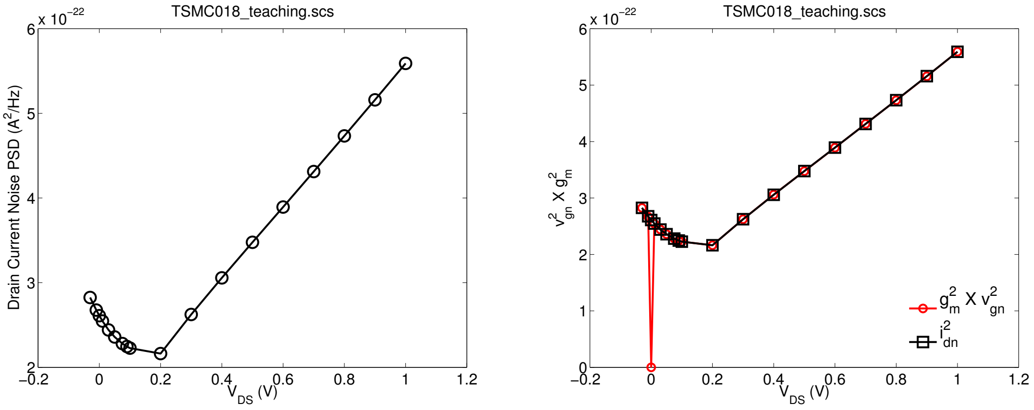
And finally considering noise contribution from both sides of LO, then

$$m \quad = \quad 2 \times \left\{ \frac{1}{2}\left(\frac{2}{\pi}\right)^2 + \frac{1}{2}\left(\frac{2}{\pi}\right)^2 \frac{1}{3^2} + \frac{1}{2}\left(\frac{2}{\pi}\right)^2 \frac{1}{5^2} + \ldots \right\} = \frac{1}{2}$$

B. *Characterizing nFET drain current noise*

When the nFET in fig. 4(b) was characterized at $V_{DS} = 0$ V, the drain current noise (spot noise at 50 MHz) was found to be 2.61×10^{-22} A^2/Hz and the output conductance of the FET (for $V_{DS} = 0$ V), $g_{d0} = 15.74$ mS. When the current noise PSD is set to $\gamma 4kTg_{d0}\Delta f$, we get $\gamma \approx 1$, which is what we expect for short-channel devices [1].

However, what is important is the channel noise when the nFET is properly biased that is non-zero V_{DS}. The channel current noise is characterized in the fig. 9(a). Fig. 9(b)



(a)

(b)

Fig. 9. Characterization of nFET channel noise and input-referred noise for TSMC018_teaching.scs

shows comparison between drain current noise PSD and the input referred voltage noise PSD multiplied by g_m^2. Apparently, only at $V_{DS} = 0$ V, they don't match since g_m is also zero there. This has been verified for Philips BiCMOS process too. So it is not a problem with the model card. In fact if you take V_{DS} close to zero instead of exactly zero, you should get the match. This again has been verified for $V_{DS} = 2$ mV for Philips process. I have not yet thought over an explanation for the shape of the noise spectral density, but the shape is maintained for Philips process too.

References

[1] Andries J. Scholten, Luuk F. Tiemeijer, Ronald van Langevelde, Ramon J. Havens, Adrie T. A. Zegers-van Duijnhoven, and Vincent C. Venezia,"Noise Modeling for RF CMOS Circuit Simulation", *IEEE Transactions on Electron Devices*, vol. ED-50, pp.618-632, Mar. 2003.