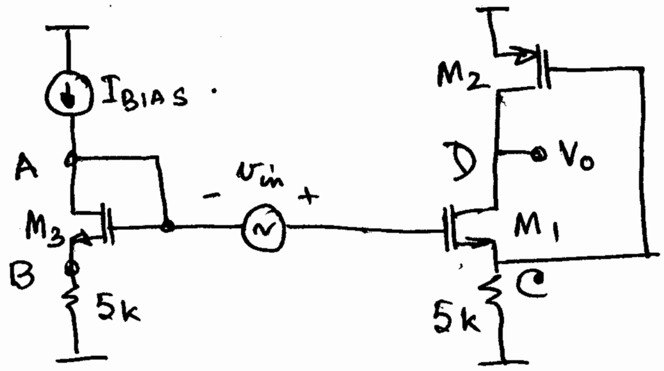
E 6312 : Advanced Analog Circuits March 10, 2004

Midterm Solutions

(1)



1) (a) DC Bias point

Voltage at B $\longrightarrow$ $5k\Omega \times 100\,\mu A = 0.5V$ [3]

Resulting voltage at A $\longrightarrow$ $\frac{100\,\mu A}{2} \times 50 \times (V_A - 0.5 - 0.5)^2$

$\approx 100\,\mu A$ [neglecting effect of V_{DS}]

$\Rightarrow V_A = \cancel{0.5}\ 1.2$ Volts

(We can use the above equation because [3]
M_1 is diode connected. $\therefore V_{GS} = V_{DS}$, $\therefore V_{DS} > V_{GS} - V_T$)
$\therefore M_1$ is in saturation.

This is a current mirror. So the same $100\,\mu A$ current flows through M_1, M_2 (if M_1 is in saturation)

$\Rightarrow$ voltage at C $= 100\,\mu A \times 5k\Omega = 0.5$ Volts. [3]

M_2 could be in saturation or linear. Let's try to find out where.

If M_2 in sat : $I_D = \frac{50\,\mu A}{2} \times 1 \times 2.0^2 \times \left(1 + \frac{V_{DS2}}{V_A}\right)$ ← for pmos

$= \frac{\cancel{50}\ 100\,\mu A}{2} \times 50 \times 0.2^2 \times \left(1 + \frac{V_{DS1}}{V_A}\right)$ ← for nmos or $\quad V_{DS_2} = V_{DS_1}$

But $V_{DS_2} + V_{DS_1} = 2.5\,V$. $\quad \therefore\ V_{DS_2} = V_{DS_1} = 1.25\,V$.

Is ~~this~~ M_2 in sat? For M_2, $\quad 1.25 \not> 2.5 - 0.5$

So the above derivation is not valid.

If M_2 in linear: $\quad I_D = 50\,\mu A \times 2.0 \times V_{DS} = 100\,\mu A$

$\therefore\ V_{DS} = 1$ Volt

Is M_2 in linear? for M_2, $\quad 1 < 2.5 - 0.5$.

So M_2 is in linear region — and $\underline{V_D = 2.0 \text{ Volts}}$

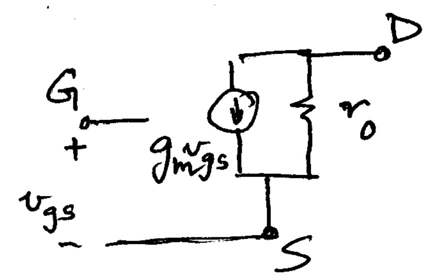
[4]

(b) $M_1 \rightarrow$ saturation, strong inv

$M_2 \rightarrow$ non-saturation, strong inv

$M_3 \rightarrow$ saturation, ~~linear~~ strong inv. [3]

(c) For any transistor M_1, M_2, M_3:



For M_1, M_3 (sat, strong inv)

$$g_{m_1} \simeq \mu C_{ox} W/L\,(V_{GS} - V_T) = 100 \times 50 \times 0.2\ \mu S = \underline{1\,mS}$$

$$g_{ds_1} = I_D / V_A = \frac{100\,\mu A}{10\,V} = 10\,\mu S$$

For M_2 (non-sat, strong inv)

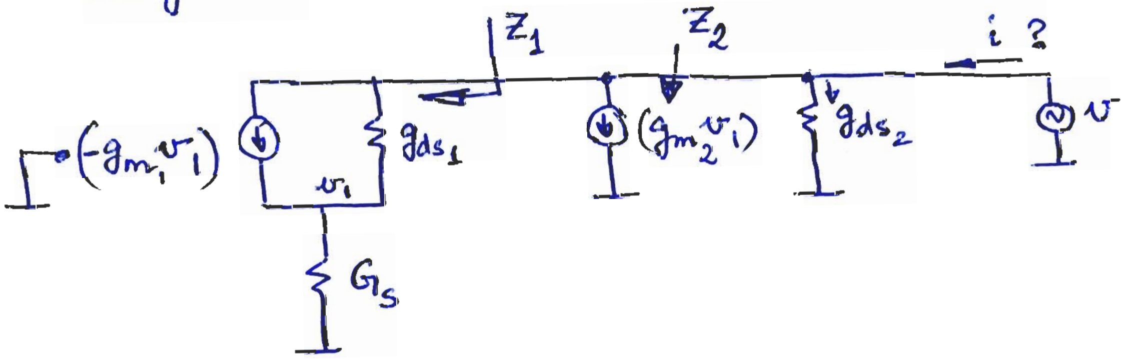
$$g_{m_2} = \mu C_{ox} \, W/L \; V_{DS} = 50 \times 1 \times 1 \; \mu S = 50 \, \mu S$$

$$g_{ds_2} = \mu C_{ox} \, W/L \, (V_{GS} - V_T) = 50 \times 1 \times 2 \; \mu S = 100 \, \mu S$$

[4]

2) <u>Small signal output impedance</u>

Apply small signal voltage at output, short input to 0, find current coming in :



[Note : Gate/drain of M_3 has a small signal voltage of 0 — always. This is because there is a constant bias current through M_3. Convince yourselves about this.]

~~Straightforward "brute force" approach~~

We do KCL at node v_1 :

$$v_1 G_S = - v_1 g_{m_1} + (v - v_1) g_{ds_1}$$

$$\therefore \; v_1 (G_S + g_{m_1} + g_{ds_1}) = v \cdot g_{ds_1}$$

$$\text{or} \quad v_1 = \frac{g_{ds_1}}{G_S + g_{m_1} + g_{ds_1}} \, v$$

Straightforward "Brute force" Approach

Now $$i = v_1 \cdot G_S + g_{m_2} v_1 + g_{ds_2} v$$

$$= v \cdot g_{ds_2} + v \cdot \frac{(g_{m_2} + G_S) \, g_{ds_1}}{G_S + g_{m_1} + g_{ds_1}} \Rightarrow G_{out} = g_{ds_2} + \frac{(g_{m_2} + G_S) \, g_{ds_1}}{G_S + g_{m_1} + g_{ds_1}}$$

$\Longrightarrow$ Plug in the values for $g_{m_1}, g_{m_2}, g_{ds_1}, g_{ds_2}$

$$R_{out} = \frac{1}{G_{out}} = \underline{\underline{9.797\ k\Omega}}. \qquad [10]$$

Alternative method :-

Looking into Z_1, we see a ~~cascode~~ device with Source degeneration of G_S.

$$Z_1 = Z_{out}\,(\text{source degeneration}) = r_{o_1} + R_S + g_{m_1} r_{o_1} R_S$$

For M_2, we see a current of $g_{m_2} v_1 + g_{ds_2} v$

$$= \left(\frac{g_{m_2} g_{ds_1}}{G_S + g_{m_1} + g_{ds_1}} + g_{ds_2}\right) v$$

$$\text{or} \quad Z_2 = \frac{1}{\dfrac{g_{m_2} g_{ds_1}}{G_S + g_{m_1} + g_{ds_1}} + g_{ds_2}}$$

$$\therefore\ 1/Z_{out} = 1/Z_1 + 1/Z_2$$

$$= \frac{1}{r_{o_1} + R_S + g_{m_1} r_{o_1} R_S} + \left(\frac{g_{m_2} g_{ds_1}}{G_S + g_{m_1} + g_{ds_1}} + g_{ds_2}\right)$$

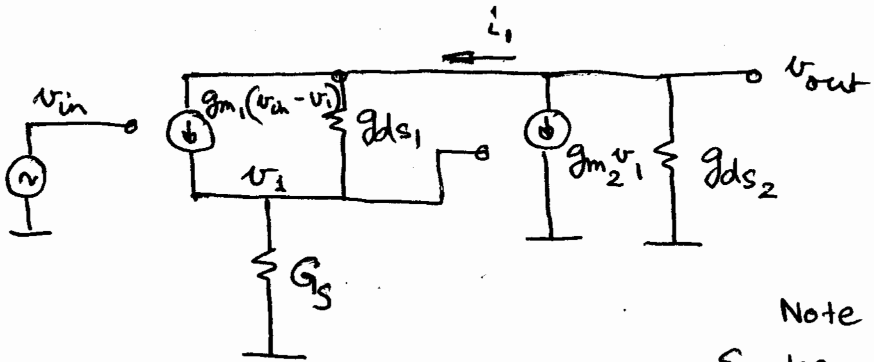
$$= \frac{G_S g_{ds_1}}{G_S + g_{ds_1} + g_{m_1}} + \frac{g_{m_2} g_{ds_1}}{G_S + g_{m_1} + g_{ds_1}} + g_{ds_2}$$

$$= g_{ds_2} + \frac{(g_{m_2} + G_S)\, g_{ds_1}}{G_S + g_{m_1} + g_{ds_1}} \Longrightarrow R_{out} = \underline{\underline{9.797\ k\Omega}}$$

(same expression as before)

3) Small Signal Gain :

"Brute force" method :



Note that $i_1 = v_1 G_S$.

So KCL at v_{out} $\Rightarrow$

$$v_1(G_S + g_{m2}) = -v_{out}\, g_{ds2}$$

or $$v_1 = -v_{out} \cdot \frac{g_{ds2}}{G_S + g_{m2}} \longrightarrow (1)$$

~~KCL for v_{out} $v_1(G_S + g_{m2}) =$~~

KCL for v_1 : $$g_{m1}(v_{in} - v_1) + g_{ds1}(v_{out} - v_1) = v_1 G_S$$

or $$g_{m1} v_{in} = -g_{ds1} v_{out} + \underset{\uparrow \text{ Plug in (1)}}{v_1}(G_S + g_{ds1} + g_{m1})$$

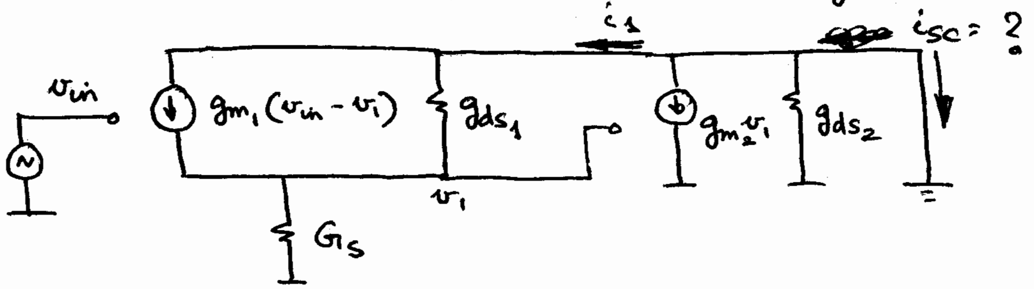
$$= -v_{out}\left(g_{ds1} + \frac{g_{ds2}}{G_S + g_{m2}}(G_S + g_{ds1} + g_{m1})\right)$$

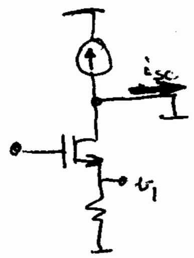
$$\Rightarrow \frac{v_{out}}{v_{in}} = -\frac{g_{m1}(G_S + g_{m2})}{g_{ds1}(G_S + g_{m2}) + g_{ds2}(G_S + g_{ds1} + g_{m1})}$$

Plug in values $\Longrightarrow$ Gain = $\underline{\underline{-2.024}}$ [10]

Another approach, using already computed Rout :

Short output to ground and measure short circuit current → This will give transconductance G_m for the circuit → gain will be $G_m R_{out}$





In this case $i_1 = v_1 G_S$.

$$-i_{sc} = i_1 + g_{m_2} v_1 = (G_S + g_{m_2}) v_1 .$$

Note that v_1 is the voltage at the source of a source follower

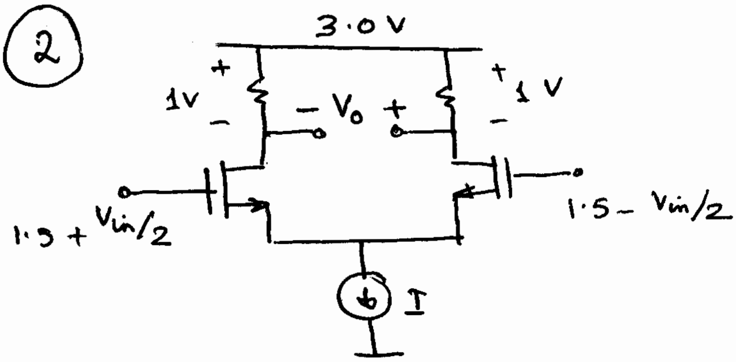
$$\text{So} \quad v_1 = \frac{1/(G_S + g_{ds_1})}{1/g_m + 1/(G_S + g_{ds_1})} \times v_{in} = \frac{g_{m_1}}{g_{m_1} + g_{ds_1} + G_S} \cdot v_{in}$$

$$\text{So} \quad i_{sc} = -\frac{g_{m_1}(G_S + g_{m_2})}{g_{m_1} + g_{ds_1} + G_S} \cdot v_{in}$$

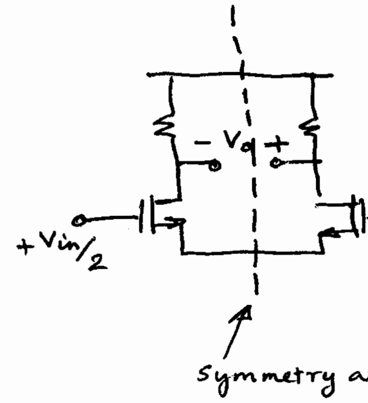
$$\Rightarrow \quad G_m = -206.61\ \mu S$$

$$\therefore \text{gain} = G_m R_{out} = \underline{\underline{-2.024}}$$

2

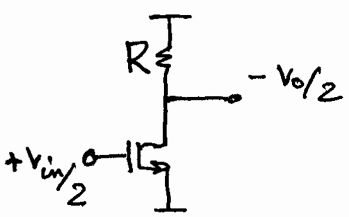


Small signal
$\Rightarrow$



In small signal, take half of the circuit

(You can do this only when the circuit is completely symmetric. Since one side has $+V_{in}/2$ and the other $-V_{in}/2$, there will be a "null" through the axis of symmetry on the circuit. This means that the source voltage will be at virtual ground and you will have $-V_o/2$ and $+V_o/2$ at the drain node.)



$\longrightarrow$

$$-V_o/2 = -g_m V_{in}/2 \cdot R$$

$$\therefore \quad V_o/V_{in} = g_m \cdot R$$

(a) Assume the device is in strong inversion. Good assumption since $V_{GS} - V_T = 200mV$

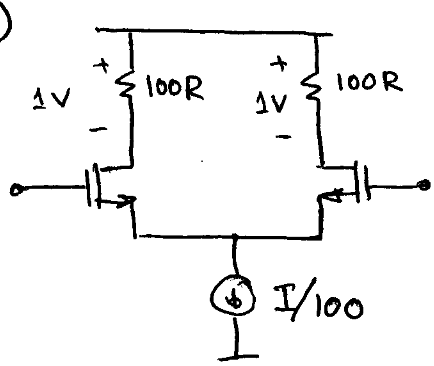
$$g_m = \frac{2I_D}{V_{GS} - V_T}$$

$$= \frac{2I_D}{200mV}.$$

$$\text{Gain} = g_m \cdot R = \frac{2I_D R}{200mV} \qquad I_D \cdot R = 1 \text{ Volt}.$$

$$= \frac{2V}{200mV} = \underline{\underline{10}}. \qquad [10]$$

(b)



Here the current through drain is 100x less than before.

For the same device, 100x lesser current, V_{GS} will have to be much lesser.

For $I/2$, $V_{GS} - V_T = 200\,mV$. $I_D \propto (V_{GS} - V_T)^2$.

If the same law holds, For $I/200$, $V_{GS} - V_T = 20\,mV$.

$\downarrow$

We know that at these voltages, the square law does not hold.

So is it in moderate inversion or weak inversion ?? Investigate further.

$I_Z = 2kn\phi_t^2$. In strong inv, $I_D = k/2\,(V_{GS} - V_T)^2$.

For current $I/2$ (prev. case) $V_{GS} - V_T = 200\,mV$, $I_D = k/2 \times 0{\cdot}04$. Inv. coeff ≈ 10

$I_Z \simeq 2k \times 1 \times 0{\cdot}3^2$

100x less current means inv. coeff is also 100x less.

So $I_D/I_Z \approx 0{\cdot}1$ $\leftarrow$ inv. coeff for this case $\Rightarrow$ this is in weak inversion.

In weak inversion, $g_m = I_D/n\phi_t = 40\,I_D$.

Gain $= g_m \cdot 100R$

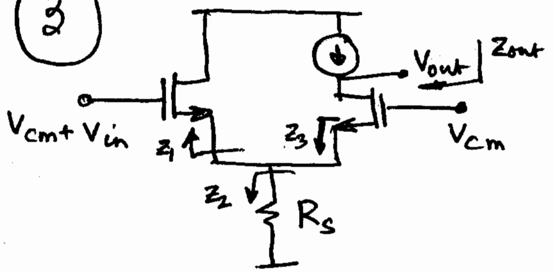
$= 40 \times 100 \times I_D \cdot R = 4000 \times I_D \cdot R$

$I_D \cdot 100R = 1$ Volt.

$\therefore$ Gain $= 40 \times (I_D \cdot 100R) = \underline{\underline{40}}$.

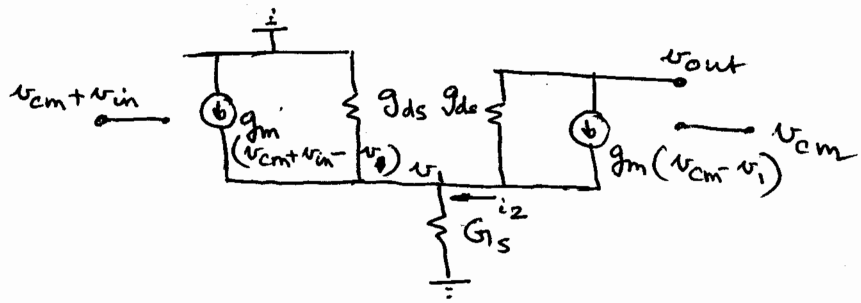
[10]

(3)



This circuit has no symmetry.
You cannot break into any half circuit.

Small signal picture :-



(1)

v_o/v_{cm} : "Brute force" method !

Set $v_{in}=0$. KCL at $v_1 \Rightarrow$ $v_1 G_s = g_m(v_{cm}-v_1) - v_1 g_{ds}$.

Note : current $i_2 = 0$ since the output is open!

$$\therefore \quad v_1 = v_{cm} \cdot \frac{g_m}{g_m+g_{ds}+G_s}.$$

$$v_{out} = v_1 - g_m(v_{cm}-v_1)/g_{ds}$$

$$= v_1(1+g_m/g_{ds}) - g_m/g_{ds}\, v_{cm}$$

$$= v_{cm}\left[\frac{g_m}{g_m+g_{ds}+G_s}\cdot\frac{g_{ds}+g_m}{g_{ds}} - \frac{g_m}{g_{ds}}\right]$$

$$= v_{cm}\cdot\frac{g_m}{g_{ds}}\left[\frac{g_m+g_{ds}}{g_m+g_{ds}+G_s} - 1\right] = -v_{cm}\cdot\frac{g_m}{g_{ds}}\cdot\frac{G_s}{g_m+g_{ds}+G_s}$$

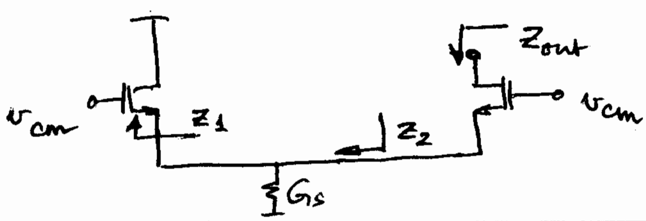
$$\therefore \quad \frac{v_{out}}{v_{cm}} = -\frac{g_m}{g_{ds}}\cdot\frac{G_s}{g_m+g_{ds}+G_s}$$

$1/g_{ds} = 10\,k\Omega$, $1/G_s = 5\,k\Omega$, $g_m = 2\,mS$

$\Rightarrow v_o/v_{cm} = \underline{\underline{-1.739}}$

[15]

More intuitive approach



Compute Z_1 : Looking in from source

$$Z_1 = \frac{1}{g_m+g_{ds}}.$$

Compute Z_2: $\quad Z_2 = Z_1 \| R_S = \dfrac{1}{g_m + g_{ds} + G_S}$

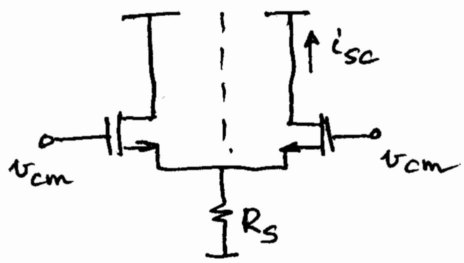
Compute Z_{out}: (source follower)

$$Z_{out} = g_m \cdot Z_2 \cdot r_o + r_o + Z_2$$

$$= (1 + g_m/g_{ds}) Z_2 + 1/g_{ds} \Rightarrow 19.13\ k\Omega$$

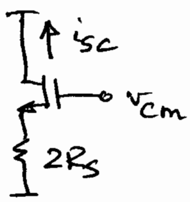
[This Z_{out} will be used in the second part too]

Now measure short circuit current:



Now, indeed this is symmetric. Break R_S into two parallel resistors of value $2R_S$. Let us break this along the dotted line. Since both sides are excited identically by v_{cm}, there will be an "open" along the symmetry a

Half circuit:



Here, $i_{sc} = -\dfrac{v_{cm}}{2R_S} \times$ gain of source follower

$$= -\frac{v_{cm}}{2R_S} \times \frac{(2R_S)\| r_o}{(2R_S)\| r_o + 1/g_m}$$

$$\Rightarrow G_{m_{CM}} = -90.9\ \mu S.$$

$\therefore \quad v_{out}/v_{cm} = G_{m_{cm}} \cdot Z_{out} = \underline{\underline{-1.739}}$.

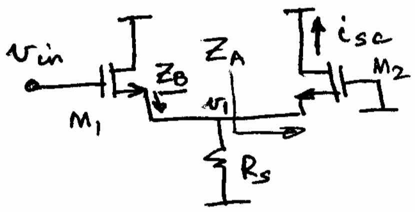
(2) "Brute force" $\quad$ set $v_{cm} = 0$. $\quad v_1 = v_{in} \cdot \dfrac{g_m}{g_m + g_{ds} + G_S}$

$$v_o = v_1 (1 + g_m/g_{ds}) = v_{in} \cdot \frac{g_m}{g_{ds}} \cdot \frac{g_m + g_{ds}}{g_m + g_{ds} + G_S}$$

$$\Rightarrow v_o/v_{in} = \underline{\underline{18.26}}$$ [15]

Intuitive approach

We have already measured Z_{out}. Let's measure i_{sc} again.



Looking into M_2 from source, $Z_A = \dfrac{1}{g_m + g_{ds}}$.

Looking at the load of M_1 at source, $Z_B = Z_A \| R_s$

$$= \frac{1}{g_m + g_{ds} + G_s}$$

So the gain from v_{in} to the source of M_1 (source follower)

$$= \frac{Z_B \| r_{o_1}}{Z_B \| r_{o_1} + 1/g_{m_1}} = \frac{\dfrac{1}{g_m + 2g_{ds} + G_s}}{\dfrac{1}{g_m + 2g_{ds} + G_s} + \dfrac{1}{g_{m_1}}} = \frac{g_m}{2g_m + 2g_{ds} + G_s}$$

i.e. voltage $v_1 = v_{in} \cdot \dfrac{g_m}{2g_m + 2g_{ds} + G_s}$

Now $i_{sc} = + g_m v_1 + g_{ds} \cdot v_1 = v_{in} \cdot \dfrac{g_m (g_m + g_{ds})}{2g_m + 2g_{ds} + G_s}$

$$\therefore \quad G_{m_{in}} = \frac{g_m (g_m + g_{ds})}{2g_m + 2g_{ds} + G_s} = 954.5\ \mu S.$$

$$\therefore \quad v_{out}/v_{in} = G_{m_{in}} \cdot \cancel{Z} \cdot Z_{out} = \underline{\underline{18.26}}$$

General Comments: For questions on small signal gain, Z_{out} etc, more than 50% marks has been awarded if you get just the circuit diagram correct. Beyond that, marks have been awarded according to work done.