1) (a) DC Bias point

Voltage at B $\rightarrow 5k\Omega \times 100\,\mu A = 0.5\,V$ \[3\]

Resulting voltage at A $\rightarrow \frac{100\,\mu A}{2} \times 50 \times (V_A - 0.5 - 0.5)^2$

$\approx 100\,\mu A$ [neglecting offset]

$\Rightarrow V_A = 0.5 + 1.2\,\text{Volts}$ if $V_{DS}$

(We can use the above equation because $M_1$ is diode connected. $\therefore V_{DS} = V_{DS}$, $\therefore V_{DS} > V_A - V_T$)

$\therefore M_1$ is in saturation.

This is a current mirror. So the same $100\,\mu A$ current flows through $M_1, M_2$ (if $M_1$ is in saturation)

$\Rightarrow$ voltage at C $= 100\,\mu A \times 5k\Omega = 0.5\,\text{Volts}$ \[3\]

$M_2$ could be in saturation or linear. Let's try to find out where.

If $M_2$ in Sat:

$\begin{align*}
I_D &= \frac{50\,\mu A}{2} \times 5 \times 2.0^2 \times \left(1 + \frac{V_{DS}}{V_A}\right) \quad \text{for pmos} \\
&= \frac{250\,\mu A}{2} \times 50 \times 0.5^2 \times \left(1 + \frac{V_{DS}}{V_A}\right) \quad \text{for nmos}
\end{align*}$
or \[ V_{DS_2} = V_{DS} \]

But \[ V_{DS_2} + V_{DS} = 2.5 \text{V} \]

\[ V_{DS_2} = V_{DS} = 1.25 \text{V} \]

Is \( M_2 \) in sat? For \( M_2 \), \[ 1.25 > 2.5 - 0.5 \]

So the above derivation is not valid.

If \( M_2 \) in linear: \[ I_D = 50 \text{mA} \times 2.0 \times V_{DS} = 100 \text{mA} \]

\[ V_{DS} = 1 \text{V} \]

Is \( M_2 \) in linear? For \( M_2 \), \[ 1 < 2.5 - 0.5 \]

So \( M_2 \) is in linear region — and \( V_D = 2.0 \text{ Volts} \)

\[ [4] \]

\[ \begin{align*}
M_1 & \rightarrow \text{saturation, strong inv} \\
M_2 & \rightarrow \text{non-saturation, strong inv} \\
M_3 & \rightarrow \text{saturation, weak strong inv.}
\end{align*} \]

\[ [3] \]

\[ \text{(c) For any transistor } M_1, M_2, M_3 : \]

\[ G = \frac{V_{DS}}{I_D} \]

For \( M_1, M_3 \) (sat, strong inv)

\[ g_{m1} = I_{DS} \times \frac{W}{L} (V_{DS} - V_T) = 100 \times 50 \times 0.2 \text{ mS} = 1 \text{ mS} \]

\[ g_{ds1} = \frac{I_D}{V_A} = \frac{100 \text{ mA}}{10 \text{ V}} = 10 \text{ mS} \]
2) Small signal output impedance

Apply small signal voltage at output, short input to 0, find current coming in:

\[ V = (\delta m_2 V_1) Z_1 + Z_2 \]

\[ i = g_{ds2} V \]

\[ g_{ds2} = \frac{g_{m2} V_{ds2}}{g_{m1} + g_{m2} + g_{ds1}} \]

Note: Gate/drain of M3 does have a small signal voltage of 0 — always. This is because there is a constant bias current through M3. Convince yourselves about this.

**Straightforward “Brute Force” Approach:**

We do KCL at node \( V_1 \):

\[ i_1 - G_M V_5 + (V_1 - V_1) g_{ds2} = 0 \]

\[ i_1 = g_{ds2} V \]

\[ i_1 = \frac{g_{ds2}}{g_{m1} + g_{m2} + g_{ds1}} \]

Now, \( i = i_1 = g_{ds2} + \frac{(g_{m2} + g_{m1}) g_{ds2}}{g_{m2} + g_{m1} + g_{ds1}} \)
Plug in the values for \( g_m, g_{m2}, g_{ds1}, g_{ds2} \)

\[
R_{out} = \frac{1}{G_{out}} = 9.797 \, k\Omega
\]

Alternative method:

Looking into \( Z_1 \), we see a cascade device with source degeneration of \( G_S \).

\[
Z_1 = Z_{out} \text{ (some degeneration)} = \frac{1}{R_2 + R_S + g_m R_S} g_m R_2 R_S
\]

For \( R_{22} \), we see a current of \( g_m v_1 + g_{ds2} v_2 \)

\[
= \left( \frac{g_{m2} g_{ds1}}{g_S + g_m + g_{ds1}} \right) v_1 + g_{ds2} v_2
\]

or \( Z_2 = \frac{1}{g_{m2} g_{ds1} + g_{ds2}} \)

\[
\frac{1}{2Z_{out}} = \frac{1}{Z_1} + \frac{1}{Z_2}
\]

\[
= \frac{1}{R_2 + R_S + g_m R_S} + \left( \frac{g_{m2} g_{ds1}}{g_S + g_m + g_{ds1}} \right)\left( \frac{g_{ds2}}{g_S + g_m + g_{ds1}} \right)
\]

\[
= \frac{G_S g_{ds1}}{g_S + g_{ds1} + g_m} + \frac{g_{ds2}}{g_S + g_m + g_{ds1}} + \frac{g_{ds2}}{g_S + g_m + g_{ds1}}
\]

\[
= \frac{g_{ds2} + \left( g_{m2} + G_S \right) g_{ds1}}{G_S + g_m + g_{ds1}} \Rightarrow R_{out} = 9.797 \, k\Omega
\]

(same expression as before)
3) Small Signal Gain:

"Brute force" method:

\[ V_{\text{out}} = \frac{g_{m_1} (V_{\text{in}} - v_i) - g_{ds_1} (V_{\text{out}} - v_i)}{g_{ds_2} + g_{m_2}} \]

Note that \( i_i = v_i G_m \).

So KCL at \( V_{\text{out}} \):

\[ v_i (G_S + g_m) = -V_{\text{out}} g_{ds_2} \]

\[ v_i = -V_{\text{out}} \frac{g_{ds_2}}{G_S + g_m} \]

KCL for \( v_i \):

\[ g_m_1 (V_{\text{in}} - v_i) + g_{ds_1} (V_{\text{out}} - v_i) = v_i G_m \]

or

\[ g_{m_1} V_{\text{in}} = -g_{ds_1} V_{\text{out}} + v_i (G_S + g_{ds_1} + g_m) \]

Plug in (1):

\[ V_{\text{out}} = -\frac{g_{m_1} (G_S + g_m)}{g_{ds_1} (G_S + g_m)} \]

\[ \Rightarrow \frac{V_{\text{out}}}{V_{\text{in}}} = -\frac{g_{m_1} (G_S + g_m)}{g_{ds_1} (G_S + g_m_2) + g_{ds_2} (G_S + g_{ds_1} + g_m)} \]

Plug in values

\[ Gain = -2.024 \]

Another approach, using already computed \( R_{\text{out}} \):

Short output to ground and measure short circuit current \( i_{SC} \). This will give transconductance \( G_m \) for the circuit \( \Rightarrow \) gain will be \( G_m R_{\text{out}} \).
In this case \( i_s = v_i G_S \).

\[-i_{sc} = i_i + g_m v_i = (G_S + g_m) v_i\]

Note that \( v_i \) is the voltage at the source of a source follower.

So \( v_i = \frac{1}{G_S + g_{ds}} \times v_{in} = \frac{g_m}{g_m + g_{ds} + g_S} \cdot v_{in} \)

So \( i_{sc} = -\frac{g_m (G_S + g_m)}{g_m + g_{ds} + g_S} \cdot v_{in} \)

\[\Rightarrow G_m = -206.61 \ \mu S\]

\(\therefore \) gain = \( G_m R_{out} = -2.024 \)
In small signal, take half of the circuit.

(You can do this only when the circuit is completely symmetric.)

Since one side has \(+V_{in}/2\) and the other \(-V_{in}/2\), there will be a "null" through the axis of symmetry on the circuit. This means that the source voltage will be at virtual ground and you will have \(-V_{in}/2\) and \(+V_{in}/2\) at the drain node.

\[
\begin{align*}
V_{in}/2 & \quad \rightarrow \quad -V_{in}/2 = -g_m \frac{V_{in}/2}{R} \\
& \quad \therefore \quad \frac{V_{in}}{V_{in}} = \frac{g_m}{R}
\end{align*}
\]

(a) Assume the device is in strong inversion. Good assumption since \(V_{DS} - V_T = 200\text{mV}\)

\[
g_m = \frac{2I_D}{V_{DS} - V_T}
\]

\[
G_{ain} = g_m \cdot R = \frac{2I_D \cdot R}{200\text{mV}} \quad I_D \cdot R = 1\text{Volt} \rightarrow \frac{2V}{200\text{mV}} = 10
\]

[10]
Here the current through drain is 100x less than before.

For the same device, 100x lesser current, $V_{GS}$ will have to be much lesser.

For $I_2$,
\[ V_{GS} - V_T = 200 \text{ mV} \]
\[ I_D = (V_{GS} - V_T)^2 \]

If the same law holds, for $I_{2/5}$,
\[ V_{GS} - V_T = 20 \text{ mV} \]

We know that at these voltages, the square law does not hold.

So is it in moderate inversion or in weak inversion? Investigate further.

\[ I_2 = 2kTn\Phi_0^2 \]

In strong inv.,
\[ I_D = \frac{kT}{q} \left( V_{GS} - V_T \right)^2 \]

For current $I_2$ (per. case),
\[ V_{GS} - V_T = 200 \text{ mV} \]
\[ I_D = \frac{kT}{q} \times 0.04 \text{ A} \]
\[ I_D = 2 \times 10^{-12} \times 0.04 \text{ A} \]

100x less current means inv. coeff is also 100x less.

So $I_{D2} < 0.1$. => inv. coeff for this case => this is in weak inversion.

In weak inversion,
\[ g_m = \frac{I_D}{n\Phi_0} = 40 I_D \]

Gain = $g_m \cdot 100R$
\[ = 4000 \times I_D \cdot R \]
\[ I_D \cdot 100R = 1 \text{ Volt} \]

\[ \text{Gain} = 40 \times (I_D \cdot 100R) = 40 \]

[10]
This circuit has no symmetry. You cannot break into any half circuit.

Small signal picture:

\[ \frac{V_{cm} + V_{in}}{R_s} \]

\[ u_{cm} = \frac{V_{cm}}{G_m + \frac{G_s}{G_d}} \]

Set \( u_{cm} = 0 \). KCL at \( u_1 \):

\[ u_1 G_s = \frac{G_m (u_{cm} - u_1)}{u_i G_d} \]

Note: current \( i_2 = 0 \) since the output is open!

\[ \frac{u_1}{u_{cm}} = \frac{G_m}{G_m + \frac{G_s}{G_d}} \]

\[ u_{cm} = u_1 - \frac{G_m (u_{cm} - u_1)}{G_d} \]

More intuitive approach:

Compute \( Z_1 \): Looking in from source

\[ Z_1 = \frac{1}{G_m + \frac{G_s}{G_d}} \]

\[ G_m = 10 \, \text{kS}, \quad G_s = 5 \, \text{kS}, \quad G_d = 2 \, \text{mS} \]

\[ \Rightarrow \frac{u_1}{u_{cm}} = \frac{1}{5.732} \]
Compute $Z_2$: 

$$Z_2 = Z_1 \parallel R_S = \frac{1}{\frac{g_m}{g_{ds}} + G_S}$$

Compute $Z_{out}$:

$Z_{out} = g_m \cdot Z_2 \cdot R_S + r_0 + Z_2$

$$= (1 + \frac{g_m}{g_{ds}}) Z_2 + \frac{1}{g_{ds}} \Rightarrow 19.13 \text{ kS}$$

[This $Z_{out}$ will be used in the second part too.]

Now measure short circuit current:

Half circuit:

$$i_{sc} = -\frac{v_{cm}}{2R_S} \times \text{gain of source follower}$$

$$= -\frac{v_{cm}}{2R_S} \times \frac{(2R_S) || r_0}{(2R_S) || r_0 + 1/G_m}$$

$$\Rightarrow G_{m,cm} = 90.9 \text{ µS}.$$ 

1. $v_{cm}/v_{cm} = G_{m,cm}$, $Z_{out} = -1.739$.

2. "Brute force" method: $v_{cm} = 0$.

$$v_0 = v_i (1 + \frac{g_m}{g_{ds}}) = v_m \cdot \frac{g_m}{g_{ds} + g_m + G_S}$$

$$\Rightarrow v_0/v_m = 18.26$$ [16]
We have already measured $Z_{out}$. Let's measure $i_{sc}$ again.

Looking into $M_2$ from source, $Z_A = \frac{1}{g_m + g_{ds}}$

Looking at the load of $M_1$ at source, $Z_B = Z_A || R_s = \frac{1}{g_m g_{ds} + g_s}$

So the gain from $v_{in}$ to the source of $M_1$ (source follower)

$$\frac{v_{out}}{v_{in}} = \frac{Z_{out} || r_s}{Z_{out} || r_s + V_{g_m}} = \frac{\frac{1}{g_m + 2g_{ds} + g_s}}{2g_m + 2g_{ds} + g_s} = \frac{g_m}{2g_m + 2g_{ds} + g_s}$$

i.e. voltage

$$v_1 = v_{in} \cdot \frac{g_m}{2g_m + 2g_{ds} + g_s}$$

Now $i_{sc} = g_m v_1 + g_{ds} v_1 = v_{in} \cdot \frac{g_m (g_m + g_{ds})}{2g_m + 2g_{ds} + g_s}$

$$G_m = \frac{g_m (g_m + g_{ds})}{2g_m + 2g_{ds} + g_s} = 954.5 \mu S$$

$$\frac{v_{out}}{v_{in}} = \frac{G_m}{Z_{out} Z_{in}} = 18.26$$

**General Comments:** for questions on small signal gain, $Z_{out}$ etc., more than 50% marks has been awarded if you got just the circuit diagram correct. Beyond that, marks have been awarded according to work done.