Random Signals and Noise
ELEN E4815
Final Examination
Columbia University

Spring Semester- 2005

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Does not include Problem #3

- Length of Examination- Three hours
- Answer All Questions

GOOD LUCK!!!
Problem #1

The power spectral density, \( P_n(f) \), of a narrow-band gaussian WSS random process, \( n(t) \), is shown below.

The process can be represented by the equation below

\[
n(t) = n_R(t) \cos 2\pi f_0 t - n_Q(t) \cos 2\pi f_0 t
\]

**a.** What is the total average power in this random process?

**b.** Find and draw the power spectral densities of the baseband processes \( n_R(t) \) and \( n_Q(t) \).

**c.** We still have the same power spectral density, \( P_n(f) \), but we now define the carrier frequency, \( f_0 \), as a different frequency, as shown on the next page.

Would the power spectral densities of \( n_{RN}(t) \) and \( n_{QN}(t) \), as defined below, change? **Explain your answer!!** If your answer is positive, show the new power spectral densities of \( n_{RN}(t) \) and \( n_{QN}(t) \).
This is the equation for \( n(t) \) but with the new carrier frequency \( f_{0N} \).

\[
n_N(t) = n_{RN}(t) \cos 2\pi f_0 t - n_{QN}(t) \cos 2\pi f_0 t
\]
Problem #2

Suppose we have an SSB-USB signal as defined below.

\[ x_{\text{SSB-USB}}(t) = s(t) \cos 2\pi f_0 t - \hat{s}(t) \sin 2\pi f_0 t \]

The function, \( s(t) \), is a perfectly bandlimited gaussian random process with the power spectral density shown, \( P_s(f) \), below.

This signal is transmitted and received by the receiver shown below.

\[
\begin{align*}
\text{n(t), } N_0/2 \text{ watts/Hz} & \quad \cos(2\pi f_0 t + \theta) \\
x_{\text{SSB-USB}}(t) & \quad H_B(f) \quad H_L(f) \quad v_{\text{out}}(t)
\end{align*}
\]

where the phase \( \theta \), is not a random variable but is not equal to zero.
a. What is the output signal, $v_{out}(t)$, for a particular value of $\theta$?

b. What is the average power at the output? Is it a function of $\theta$?

c. Is the output signal, $v_{out}(t)$, a WSS signal?

   Explain your answer. If your answer is positive what is its power spectral density?

d. If $\theta$ is a random variable, with a uniform density function from $-\pi$ to $\pi$, would your answers to parts (b) and (c) above change. Again Explain your answer!!

END OF THE EXAMINATION!!!