Random Signals and Noise  
ELEN E4815  
Columbia University  
Spring Semester- 2006  
Prof I. Kalet  
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Midterm Examination  

- Length of Examination- Two Hours  
- Answer All Questions  

Good Luck!!!
Problem #1 (34 Points)

The autocorrelation function, \( R_x(\tau) \), of a random process, \( x(t) \), is shown below.

\[ R_x(\tau) \]

-3T  -2T  -T  0  T  2T  3T  \( \tau \)

a) What is the total power in this random process?

b) Is there a d-c component in this random process? Explain your answer.

c) If your answer to part (b) was positive, how much power is there in the d-c component of the random process, \( x(t) \)?

d) Find and draw the power spectral density of this random process.
e) Now suppose that the power spectral density of a WSS random process, \( x(t) \), is given below.

What is the total power in this random process?

f) Is there a d-c component in this random process? Explain your answer.
**Problem #2 (33 points)**

Consider the random process, \( x(t) \), the so-called random telegraph signal (shown below).

In this signal, which started at \(-\infty\) and will continue to \(+\infty\), the voltage flips back and forth, between \(+A\) volts and \(+A/4\) volts, in the following manner.

The switching times are dictated by a Poisson distribution, i.e., the probability of “\( k \)” switches in \( \tau \) seconds is given by the Poisson distribution function shown below.

\[
\text{Prob \{of “} k \text{” switches in } \tau \text{ seconds \} = } \frac{e^{-\lambda \tau} \{ \lambda \tau \}^k}{k!}
\]

where “\( k \)” = 0, 1, 2, 3, …

\( x(t) \)

\( A \)

\( A/4 \)

\( 0 \)

\( t \)

This is “a switching instant”
a) Find and draw the autocorrelation function of this random process-THINK!!!
b) Find and draw the power spectral density of the random process described above.
c) What is the average power of this random process?
Problem #3 (33 points)

You are given the following random process, \( x(t) \), defined by the equation below

\[
x(t) = A \cos 2\pi Wt + B \sin 2\pi Wt \quad -\infty < t < \infty
\]

where both \( A \) and \( B \) are independent random variables with average values equal to zero, i.e., \( E\{A\} = E\{B\} = 0 \), and \( E\{A^2\} = E\{B^2\} = \sigma^2 \).

a) Is \( x(t) \) a wide-sense stationary process? Explain your answer!!

b) If \( E\{A\} = E\{B\} = K \) where \( K \neq 0 \), does your answer to part (a) change? Explain your answer!!

c) The random process, \( x(t) \) defined below is a wide-sense stationary process.
Is it ergodic in the mean? Explain your answer
Is it ergodic in the autocorrelation function? Explain your answer!

\[
x(t) = A \quad \text{for} \quad -\infty < t < \infty. \quad \text{With probability}=1/3
\]
or
\[
x(t) = -A \quad \text{for} \quad -\infty < t < \infty. \quad \text{With probability}=1/3
\]
or
\[
x(t) = 0 \quad \text{for} \quad -\infty < t < \infty. \quad \text{With probability}=1/3
\]

The End!!!
Some Helpful Equations

- $\cos x \cos y = \frac{1}{2} \left[ \cos (x-y) + \cos(x+y) \right]$
- $\sin x \sin y = \frac{1}{2} \left[ \cos (x-y) - \cos(x+y) \right]$

-A periodic function, $z(t)$, may always be expanded, using the Fourier Series, to

$$z(t) = \sum c_n e^{j2\pi n(t/T)} \quad -\infty < n < \infty$$

$$n=\ldots-3,-2,-1,0,1,2,3,\ldots$$

Where $c_n$ are the Fourier Coefficients of the periodic function, $z(t)$.

If you need this result for an answer to one of the questions, you may leave answer as a function of $c_n$!!!