Problem #1

In class we mentioned that there were very few FM signals for which we could actually find the autocorrelation functions. One of the exceptions to the rule, is a signal, which is phase modulated by a gaussian random process.

Assume that we now have the phase modulated signal \( x(t) \) given by the equation below.

\[
x(t) = A \cos [2\pi f_0 t + \phi(t) + \theta]
\]

The modulation signal, \( \phi(t) \), is a WSS gaussian random process with \( \mathbb{E}\{\phi(t)\} = 0 \), and autocorrelation function, \( R_\phi(\tau) \).

The phase \( \theta \), is a random variable with a uniform probability density function between \(-\pi\) and \(\pi\).
If the function, $\phi(t)$, shown in the equation below, is given by the integral,

$$
\phi(t) = 2\pi h \int_{-\infty}^{t} a(t) \, dt,
$$

then $x(t)$ is an FM signal.

Show that the autocorrelation function, $R_x(\tau)$, of $x(t)$, is given by the equation below

$$
R_x(\tau) = \left(\frac{A^2}{2}\right) e^{-R_x(0)} e^{R_x(\tau)} \cos 2\pi f_0 \tau
$$

**Hint**: Try using the complex representation of the signal $x(t)$ and then try using the moment generating or characteristic functions for a two dimensional gaussian random variable.

This is one of the few examples for which we can actually find the autocorrelation of an FM signal.

The Power Spectral Density would still be a problem!