Modern Digital Modulation Techniques
ELEN E6909
Columbia University
Spring Semester 2008

PROBLEM SET # 8

Due Date: 22 April 2008

Problem #1

a) The S/N (=P/N_0W) ratio at the receiver, on the old analog telephone lines was designed to be above 28 dB in a bandwidth of about 3 kHz. Find the minimum capacity (using the 28 dB) for this channel, using Shannon's capacity equation.

b) For the same P/N_0 ratio used in part (a) find the capacity of the telephone line if the bandwidth had been infinite and not 3 kHz? How much capacity could have been gained, compared to the 3 kHz channel, by having an infinite bandwidth channel?

c) Find how much bandwidth is required to achieve 99% of the capacity of the infinite bandwidth channel, for the value of P/N_0 used in part (b)?

Discuss the results above , in terms of the importance of bandwidth and power in determining channel capacity for a real channel?

Problem #2

The example below is meant to give us a heuristic understanding of the BLAST (or MIMO) concept for Rayleigh fading channels.
We will however assume in the rest of the example that the channels are ordinary AWGN channels (no fading!!!).

We will compare two systems, System A and System B, which are similar to SIMO and MIMO systems.

The classic capacity equation of Shannon refers to a perfectly bandlimited ideal- no-fading –channel.

The famous equation for capacity is given below:

For one receiver and one transmitter (SISO-no fading)

$$C \leq W \log_2 [1 + P/N_0W] \text{ bps}$$

System A (SIMO)

Now imagine that we have one transmitting antenna, but N_R receiving antennas, and that the received signals at all the antennas are combined optimally, by adding up all the received powers coherently.

Therefore, the capacity for this new system will simply be given by

For one Transmitter – and N_R independent receiving antennas

$$C \leq W \log_2 [1 + (N_R P) / N_0 W] \text{ bps}$$

System B (MIMO)

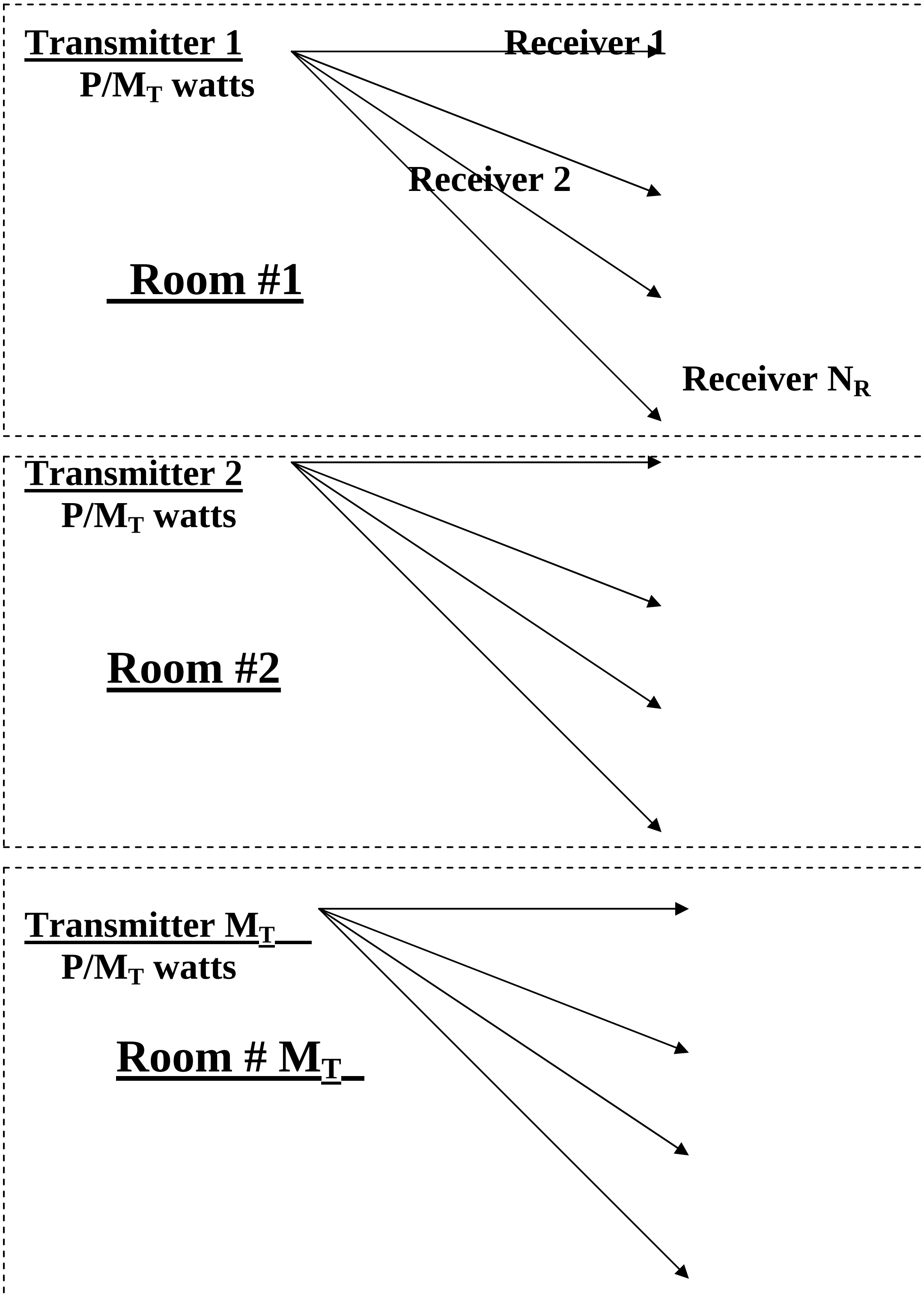
We have M_T independent transmitting antennas (transmitting independent information), each one transmitting P/M_T watts (the total transmitted power which was originally P_T, for one transmitting antenna, is now divided up evenly amongst the M_T transmitters).

Imagine that each one of the transmitting antennas is in a separated isolated room, and that in each room there are N_R independent receiver antennas for each of the transmitting antennas.

We assume that all rooms are completely isolated so that no transmitted signal interferes with the room of any other transmitting antenna.

SEE DIAGRAM ON NEXT PAGE

For each independent transmitter, there are N_R independent receivers.



In <u>each individual room</u>, we are transmitting P/M_T watts and using N_R receivers so the capacity in each individual room would be

$$C \leq W \log_2 [1 + N_R (P/M_T) / N_0 W] \text{ bps}$$

The total capacity for all rooms would just be equal to

$$C_{Total} \leq M_T \{W \log_2 [1 + N_R (P/M_T) / N_0 W]\} \text{ bps}$$

a. Find the capacity of system A, if $P/N_0W \gg 1$.

b. Find the capacity of system B if $P/N_0W \gg 1$.

<u>Now assume that $M_T=N_R=N$</u>

c. Compare the capacity of the two systems when <u>$P/N_0W \gg 1$</u>.

Show that in the MIMO case (System B) the capacity is a <u>linear function</u> of the number of antennas, N, and that in the SIMO case (System A) the capacity is a <u>logarithmic function</u> of the number of antennas, N. This example is very similar to the example of BLAST for Rayleigh fading channels. In the example of BLAST or MIMO, it is the Rayleigh fading which creates the equivalent of the individual rooms with no interference.

d.Now repeat parts a, b and c if <u>$P/N_0W \ll 1$.</u>

What can you say about MIMO when <u>$P/N_0W \ll 1$?</u>