

Modern Digital Modulation Techniques
ELEN E6909
Columbia University
Spring Semester 2008

PROBLEM SET # 6 (New Set Number)

Due Date: 9 April 2008

Read the following articles on BLAST and MIMO

The theoretical background behind MIMO.

1. G.J. Foschini and M.J. Gans, “On limits of wireless communications in a fading environment when using multiple antennas”, Wireless Personal Communications, Vol. 6, No. 3, 1998, pp. 311-335.

The BLAST Algorithm

2. P. W. Wolniansky, G. J. Foschini, G. D. Golden, R. A. Valenzuela, “**V-BLAST: An Architecture for Realizing Very High Data Rates Over the Rich-Scattering Wireless Channel**”, Invited Paper, Proc. ISSSE-98, Pisa, Italy, Sept. 29, 1998.

These articles may be downloaded from the following website

<http://www1.bell-labs.com/project/blast/>

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Problem #1

**This problem concerns Maximal Ratio Combining
(MRC) –SIMO techniques**

a) The modulation technique is BPSK. Find the outage probability at the output of a maximal-ratio combining receiver (with two receiving antennas) as a function of the average received energy per bit per antenna, divided by the noise spectral density, $E_{b, \text{avg, ant}}/N_0$, and the required instantaneous, $E_{b, \text{req, ant}}/N_0$, for the required instantaneous probability of error. Assume that the receiving antennas receive independent signals of the same average power.

b) Now find the outage probability of a maximal-ratio combining receiver (with two receiving antennas) as a function of the total average received energy per bit at both antennas divided by the noise spectral density, $E_{b, \text{avg, total}}/N_0$. Assume that the antennas receive independent signals of the same average power.

c) For BPSK, compare the results of (a) and (b) with those for a single receiving antenna at an outage probabilities, of 10^{-3} and 10^{-1} , if the desired instantaneous $\Pr_b\{\varepsilon\}=10^{-5}$. **How many dB have been gained in each case by using MRC-SIMO techniques?**

Problem #2

This problem concerns MRC techniques when the number of receiving antennas is “L”.

For BPSK, show that the probability density function, $f(x)$, for the combined received signal for L antennas, with maximal-ratio combining, is given by the equation below.

$$f(x) = \frac{1}{(L-1)! (2\sigma^2)^L} x^{L-1} \exp\{-x/2\sigma^2\}; \quad x \geq 0$$

(where $x = x_1 + x_2 + \dots + x_L$) ; $x_i = r_i^2$.

The variable, r_i represents the random Rayleigh variable at each receiving antenna.

Hint: This is similar to what we did in class for two receiving antennas.