Modern Digital Modulation Techniques ELEN E6909 Columbia University Spring Semester 2008

PROBLEM SET # 3

Due Date: 19 February 2008

This is really a review assignment.

Review the material on Nyquist signaling which you have covered in earlier courses. I have including some figures on the subject, on the website, which may help you.

Also I have listed reading assignments below, on the subject, in Haykin's book.

| • NYQUIST SIGNALS | |
|--------------------------------|----------|
| -Nyquist Signals | H259-264 |
| -Eye Pattern | |
| -Nyquist I Theorem | H259-264 |
| -Nyquist Signals with | H264-267 |
| Raised Cosine Filtering | |
| -Duobinary Signaling | H267-275 |
| | |

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PROBLEM SET # 4

Due Date: 26 February 2008

Problem #1.

a. Show that the general equation for <u>the average energy, E_s </u>, of a <u>square</u> QAM constellation is given by

 $E_s = [(M-1)/6] d_{min}^2$

where M=2ⁿ and n=number of bits/symbol

b. Find the peak-to-average energy ratio of a general square QAM constellation. The peak-to-average energy ratio, in this case, is defined as the ratio of the energy of the strongest signal in the signal set divided by the average energy of the constellation.

c. Find the peak-to-average ratio defined in (b) as M becomes very large.

d. Find a general expression for the Probability of Symbol Error, $Pr_s{\epsilon}$, for an M-ary Pulse Amplitude Modulation (PAM).

Problem # 2

We are trying to design a new 16 QAM signal set, which has a smaller peak-to-average ratio than a square 16 QAM constellation.

We would like d_{min} to be the same in both constellations (This implies that the probability of symbol error is approximately the same for both cases).

Our new signal set is made up of two rings of 8-PSK.

The minimum distance, between points in the inner ring, is equal to d_{min} . The second (or outer) ring of points is rotated by 22 ½ degrees from the first ring and the radius of the second (outer) ring is chosen so that the distance from any point on the outer ring to the nearest points on the inner ring is the same d_{min} that we used earlier for the inner ring.

- a) Find the average energy per signal, E_s , for the new QAM set as a function of d_{min} and compare to the average energy of the original square constellation for the same d_{min} . Which set requires less energy?
- b) Compare the peak-to-average energy ratios for both sets. Which set has the smaller peak-to-average ratio?



Problem # 3

Prove that the probability of symbol error, $Pr_s{\epsilon}$, for Orthogonal MFSK is given by the equation below. Most of the proof will be done in class.

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\Pr_{s}\{\epsilon\}=1-\int_{-\infty}^{\infty} [1/\sqrt{2\pi}] \{\exp[-(y-\sqrt{2E_{s}}/N_{0})^{2}/2]\} [1-Q(y)]^{M-1} dy.
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Problem # 4.

We are considering an Orthogonal MFSK system operating over an additive white gaussian noise (AWGN) channel.

a. If we want the symbol error probability to be equal to 10^{-5} , what value of E_b/N_0 in dB is required, for values of M equal to 2,4,8,16,64 and 256.

(Use the upper bound which we found in class for the $Pr_s{\epsilon}$.)

b. Show that the bit error rate, $Pr_b{\epsilon}$, for an Orthogonal MFSK modulation is equal to [(M/2)/(M-1)] times the $Pr_s{\epsilon}$, the symbol error rate.

c. For Orthogonal MFSK, what is the $Pr_b{\epsilon}$ as a function of $Pr_s{\epsilon}$, for large values of M?