# Modern Digital Modulation Techniques ELEN E6909 Columbia University Spring Semester 2008 

## PROBLEM SET \# 3

Due Date: 19 February 2008
This is really a review assignment.
Review the material on Nyquist signaling which you have covered in earlier courses. I have including some figures on the subject, on the website, which may help you.

Also I have listed reading assignments below, on the subject, in Haykin's book.

- NYQUIST SIGNALS
-Nyquist Signals
H259-264
-Eye Pattern
-Nyquist I Theorem
H259-264
-Nyquist Signals with
Raised Cosine Filtering
-Duobinary Signaling
H267-275


# Modern Digital Modulation Techniques <br> ELEN E6909 <br> Columbia University Spring Semester 2008 

## PROBLEM SET \# 4

Due Date: 26 February 2008

## Problem \#1.

a. Show that the general equation for the average energy, $\mathrm{E}_{\mathbf{s}}$, of a square QAM constellation is given by

$$
E_{s}=[(M-1) / 6] d_{\min }^{2}
$$

where $M=\mathbf{2}^{\mathrm{n}}$
and $n=$ number of bits/symbol
b. Find the peak-to-average energy ratio of a general square QAM constellation. The peak-to-average energy ratio, in this case, is defined as the ratio of the energy of the strongest signal in the signal set divided by the average energy of the constellation.
c. Find the peak-to-average ratio defined in (b) as $\mathbf{M}$ becomes very large.
d. Find a general expression for the Probability of Symbol Error, $\operatorname{Pr}_{\mathrm{s}}\{\varepsilon\}$, for an $\mathbf{M}$-ary Pulse Amplitude Modulation (PAM).

## Problem \# 2

We are trying to design a new 16 QAM signal set, which has a
smaller peak-to-average ratio than a square 16 QAM
constellation.
We would like $d_{\text {min }}$ to be the same in both constellations (This implies that the probability of symbol error is approximately the same for both cases).

Our new signal set is made up of two rings of 8-PSK.
The minimum distance, between points in the inner ring, is equal to $d_{\text {min }}$. The second (or outer) ring of points is rotated by $221 / 2$ degrees from the first ring and the radius of the second (outer) ring is chosen so that the distance from any point on the outer ring to the nearest points on the inner ring is the same $d_{\text {min }}$ that we used earlier for the inner ring.
a) Find the average energy per signal, $\mathrm{E}_{\mathrm{s}}$, for the new QAM set as a function of $d_{\text {min }}$ and compare to the average energy of the original square constellation for the same $d_{\text {min }}$. Which set requires less energy?
b) Compare the peak-to-average energy ratios for both sets. Which set has the smaller peak-to-average ratio?


## Problem \# 3

Prove that the probability of symbol error, $\operatorname{Pr}_{s}\{\varepsilon\}$, for Orthogonal MFSK is given by the equation below. Most of the proof will be done in class.

$$
\operatorname{Pr}_{\mathrm{s}}\{\varepsilon\}=1-\int_{-\infty}^{\infty}[1 / \sqrt{2 \pi}]\left\{\exp \left[-\left(\mathrm{y}-\sqrt{2 \mathrm{E}_{\mathrm{s}} / \mathrm{N}_{0}}\right)^{2} / 2\right]\right\}[1-\mathrm{Q}(\mathrm{y})]^{\mathrm{M}-1} \mathrm{dy}
$$

## Problem \# 4.

We are considering an Orthogonal MFSK system operating over an additive white gaussian noise (AWGN) channel.
a. If we want the symbol error probability to be equal to $10^{-5}$, what value of $E_{b} / N_{0}$ in $d B$ is required, for values of $M$ equal to $\mathbf{2 , 4 , 8 , 1 6 , 6 4}$ and 256.
(Use the upper bound which we found in class for the $\operatorname{Pr}_{s}\{\varepsilon\}$.)
b. Show that the bit error rate, $\operatorname{Pr}_{b}\{\varepsilon\}$, for an Orthogonal MFSK modulation is equal to [(M/2)/(M-1)] times the $\operatorname{Pr}_{s}\{\varepsilon\}$, the symbol error rate.
c. For Orthogonal MFSK, what is the $\operatorname{Pr}_{b}\{\varepsilon\}$ as a function of $\operatorname{Pr}_{s}\{\varepsilon\}$, for large values of $M$ ?

