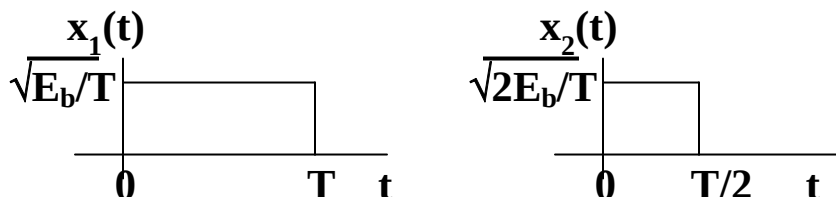


Modern Digital Modulation Techniques
ELEN E6909
Columbia University
Spring Semester 2008

PROBLEM SET # 2
Due Date: 12 February 2008

1. We are given the pair of equal energy signals shown below.



a) Find two orthonormal basis functions (Remember the Gram-Schmidt Procedure), such that $x_1(t)$ and $x_2(t)$ may be written as

$$x_1(t) = a_1\phi_1(t) + a_2\phi_2(t)$$

$$x_2(t) = b_1\phi_1(t) + b_2\phi_2(t)$$

*Use $\phi_1(t) = x_1(t) / \sqrt{E}$

*You may also use the correlation coefficient, ρ , defined as

$$\rho = \frac{\int_0^T x_1(t) x_2(t) dt}{\sqrt{E_1 E_2}} = (x_1(t), x_2(t)) / \sqrt{E_1 E_2}$$

What is $\phi_2(t)$ as a function of ρ and $\phi_1(t)$?

b) Find the coefficients a_i , b_i as functions of E and ρ .

Problem # 2

Prove the statement which was made in class, during the review, that the distance, $d_{x,y}$, between two signals, $x(t)$ and $y(t)$, defined in the time domain, by the equation below, equals the distance between the two vectors, “a” and “b” representing the two signals.

$$d_{x,y}^2 = \int_0^T (x(t) - y(t))^2 dt$$

$$d_{a,b}^2 = |a - b|^2$$

Problem # 3

The equation below is the equation describing the optimum decision rule in signal space.

If the inequality is true, we decide on signal “1”, and if the reverse is true we decide on signal ‘0’.

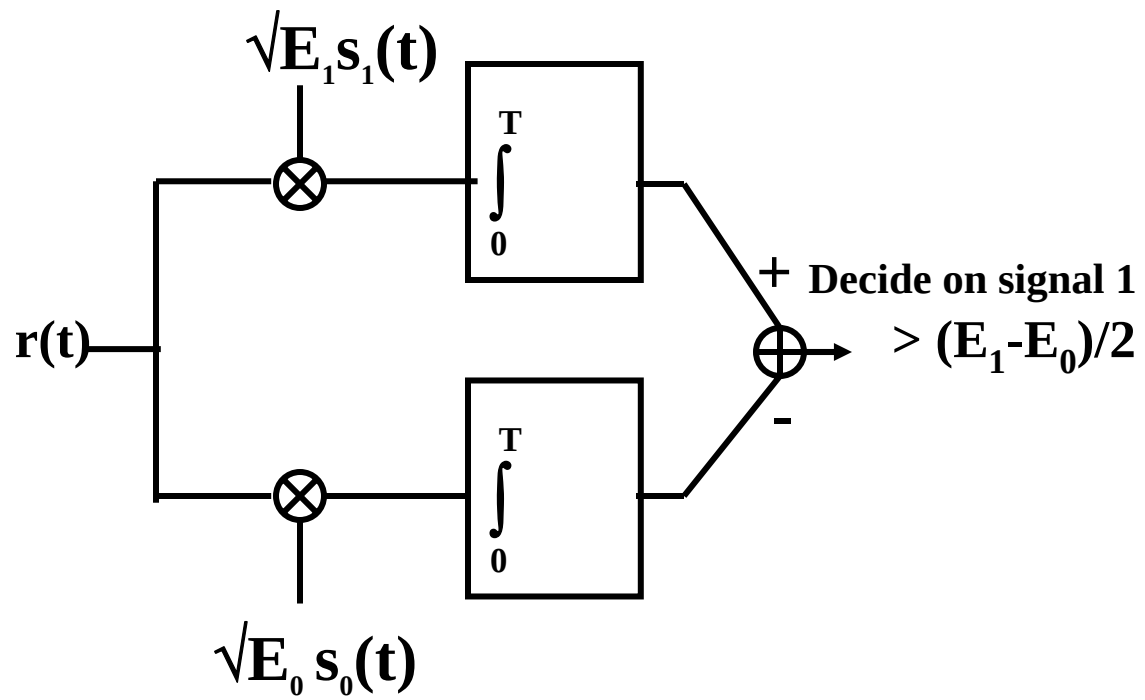
$$\frac{\int_0^T [r(t) - E_1 s_1(t)]^2 dt}{\int_0^T [r(t) - E_0 s_0(t)]^2 dt} < 1$$

In inner product form, it is written as the equation below

$$\| r(t) - \sqrt{E_1} s_1(t) \|^2 < \| r(t) - \sqrt{E_0} s_0(t) \|^2$$

Using the equation above in the time domain, show that the resulting optimum receiver has the form which we drew in class and, which is shown again on the next page.

THE CORRELATION RECEIVER



Problem # 4

You are given a binary communications system, using the two signals shown below operating over an additive white gaussian noise channel ($N_0/2$ watts /Hz).

a) Find and draw the optimum two-branch correlation receiver

b) What is the probability of bit error for this receiver?

c) Draw the two-branch matched filter for this modulation technique.

d) What is the probability of bit error for this receiver?

