# Modern Digital Modulation Techniques ELEN E6909 <br> Columbia University Spring Semester 2008 

## PROBLEM SET \# 1

Due Date: 5 February 2008

1. a. Find the average value, the variance and the average of the square of the random variable, $r$, and the Cumulative Probability Distribution Functions, for the two important (for wireless communications) probability densities shown below.

- Rayleigh probability density function

$$
f(r)=\left(r / \sigma^{2}\right) \exp \left\{-r^{2} / 2 \sigma^{2}\right\} \quad ; r \geq 0
$$

## - Rician probability density function

$$
\begin{aligned}
& f(r)=\left(r / \sigma^{2}\right)\left(\exp \left[-\left\{r^{2}+A^{2}\right\} / 2 \sigma^{2}\right]\right) I_{0}\left[r\left(A / \sigma^{2}\right)\right] \quad ; r \geq 0 \\
& I_{0}[x]=(1 / 2 \pi) \int_{-\pi}^{\pi} e^{x \cos \theta} d \theta
\end{aligned}
$$

The variance and the second-moment of the Rician random variable may be left in integral form- See if you can identify the integral, from standard probability books. These density functions are very important in wireless communications.
b. Find the probability density and cumulative distribution functions for $x$, if $x=r^{2}$ and $r$ is a Rayleigh random variable. Draw $f(r)$ and $f(x)$.

