Modern Digital Modulation Techniques ELEN E6909 Columbia University Spring Semester 2008

> **PROBLEM SET # 1 Due Date: 5 February 2008**

- 1. a. Find the <u>average value</u>, the variance and the average of <u>the square of the random variable</u>, <u>r</u>, and the <u>Cumulative</u> <u>Probability Distribution Functions</u>, for the two important (for wireless communications) probability densities shown below.
- Rayleigh probability density function

 $f(r) = (r/\sigma^2) \exp\{-r^2/2\sigma^2\}$; $r \ge 0$

• Rician probability density function

$$f(\mathbf{r}) = (\mathbf{r}/\sigma^{2}) (\exp[-\{\mathbf{r}^{2} + \mathbf{A}^{2}\}/2\sigma^{2}]) I_{0}[\mathbf{r}(\mathbf{A}/\sigma^{2})] \quad ;\mathbf{r} \ge 0$$
$$I_{0}[\mathbf{x}] = (1/2\pi) \int_{-\pi}^{\pi} e^{\mathbf{x}\cos\theta} d\theta$$

The variance and the second-moment of the Rician random variable may be left in integral form- See if you can identify the integral, from standard probability books. These density functions are very important in wireless communications.

b. Find the probability density and cumulative distribution functions for x, if $x=r^2$ and r is a Rayleigh random variable. Draw f(r) and f (x).