Power Laws in ALOHA Systems

E6083: lecture 7

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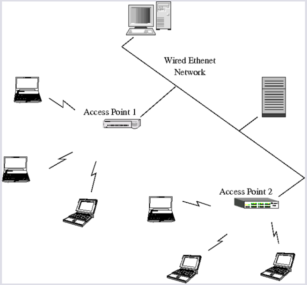
Outline

Multiple Access

Nodes share a common medium

- A receiver can hear several transmitters
- A transmitter can broadcast to several receivers

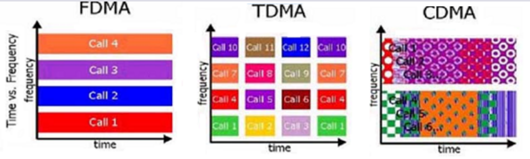


The major problem with multi-access is to decide how to share the medium when the nodes do not know if the other nodes have data to send.

Approaches to Multiple Access

Fixed Assignment

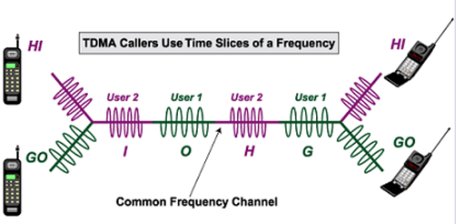
TDMA, FDMA, CDMA: each node is allocated a fixed fraction of bandwidth, very inefficient for light traffic and complex

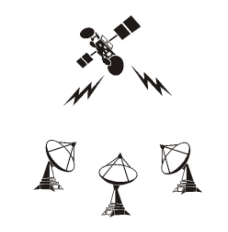


Contention systems

- Random access (ALOHA)
- Reservation
- Polling

Fixed Assignment VS. Contention





Random Access Protocols

- When a user has data to send, she
  - transmits at full channel data rate C.
  - does not coordinate with other users
- Two or more transmitting nodes cause “collision”.
- Random access MAC protocol specifies:
  - collision detection
  - recovery from collisions (delayed retransmissions)
- Examples of random access MAC protocols:
  - slotted ALOHA
  - unslotted ALOHA
  - CSMA

ALOHA

A family of contention-type Multiple access protocols

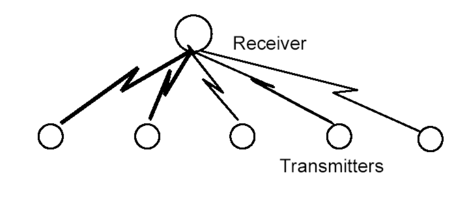


Figure: Satellite system, wireless

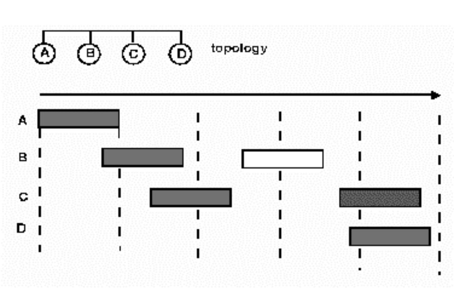
Two basic types of ALOHA systems

- Pure (unslotted) ALOHA
- Slotted ALOHA

Pure ALOHA

Basic version of ALOHA protocols

- Poisson external arrival (infinite population model)
- New packets are transmitted immediately, no synchronization
- Immediate feedback
- If no conflict, packet will be transmitted correctly
- If two or more packet transmissions overlap in time, none is transmitted successfully and need retransmission



Simple Calculation of the Throughput of Pure ALOHA

If the length of the packets is fixed T, and the rate of scheduled packets is g, then a packet sent at time t is successful if and only if no other packet is scheduled in the interval $(t - T, t + T)$, thus

$$\mathbb{P}_{suc} = e^{-2gT},$$

Throughput is

$$S = gTe^{-2gT}.$$

The throughput achieves maximum $S_{max} = \frac{e}{2}$ when $gT = \frac{1}{2}$.

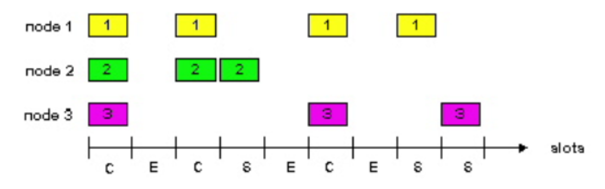
Assumptions of Slotted ALOHA

- Poisson external arrivals with rate λ
- No capture
  - Packets involved in a collision are lost
  - Capture models are also possible
- Immediate feedback
- If a new packet arrives during a slot, transmit in next slot (or assume arrivals happen at the beginning of each slot)
- If a transmission has a collision, node becomes backlogged - while backlogged, transmit in each slot with probability q_r until successful.
- No buffering at each node (queue size $= 1$).

The assumption of constant transmission rate g is flawed...

Slotted ALOHA

- Time is divided into slots of one packet duration $T = 1$
- New packet will be send in the beginning of the next slot
- Successful only if exactly one packet is scheduled for transmission during the slot
- In case of collision, node becomes backlogged



Instability of Slotted ALOHA

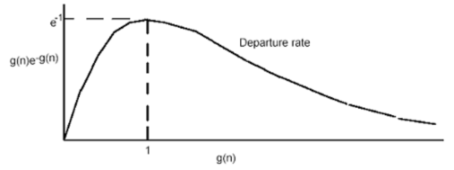
- Let $g(n)$ be the attempt rate (the expected number of packets transmitted in a slot) in state n,

$$g(n) = \lambda + nq_r.$$

- The number of attempted packets per slot in state n is approximately a Poisson random variable of mean $g(n)$.
  - $\mathbb{P}[\text{ m attempts}] = g(n)^m e^{-g(n)}/m!$
  - $\mathbb{P}$[idle] = probability of no attempts in a slot $= e^{-g(n)}$
  - $\mathbb{P}$[success] = probability of one attempt in a slot $= g(n)e^{-g(n)}$
  - $\mathbb{P}[\text{collision}] = \mathbb{P}[\text{two or more attempts}] = 1 - \mathbb{P}[\text{idle}] - \mathbb{P}[\text{success}]$

Throughput of Slotted ALOHA

$$\mathbb{P}[\text{success}] = g(n)e^{-g(n)}$$

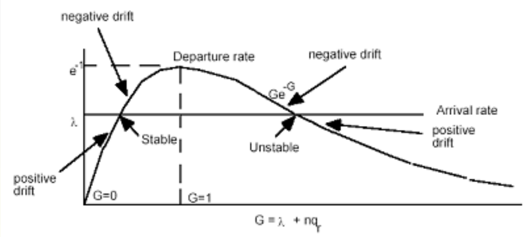


$g(n) = 1$ maximizes the throughput

$$\mathbb{P}[\text{success}] = 1/e \approx 0.36$$

Instability of Slotted ALOHA with Infinite Number of Users

- If backlog increases beyond unstable point, then the departure rate will drop to 0.
- Drift $= \lambda - \mathbb{P}[\text{success}]$.

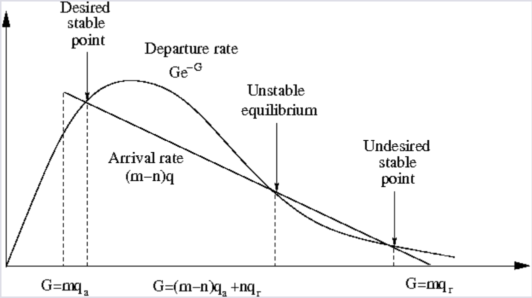


Stabilizing Slotted ALOHA

- Since $g(n) = \lambda + nq_r$, choosing q_r small increases the backlog at which instability occurs, but also increases delay.
- Method: estimate the backlog from past feedback Given the backlog estimate, choose q_r to keep $g(n) = 1$. A good rule is increase the estimate of n on each collision, and to decrease it on each idle slot or successful slot.
- For more detailed discussion of ALOHA/MAC protocols see the standard textbook "Data Networks" by D. Bertsekas and Gallager, Prentice Hall, 2nd edition, 1992.

Slotted ALOHA with Finite Users

- n is the total number of users
- q_a is the arrival probability
- q_r is the retransmission probability with $q_r > q_a$



Implies that the throughput is positive for slotted ALOHA with finite users. However, our findings challenge the traditional result.

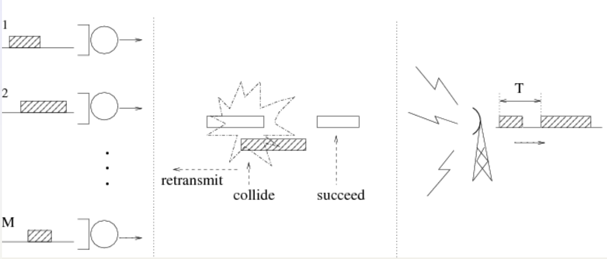
ALOHA with Variable Size Packets: Model Description

The rest of this and the following lecture is based on our paper:

P. R. Jelenkovic and J. Tan, "Is ALOHA Causing Power Law Delays?", ITC-20, Ottawa, Canada, June 17-21, 2007, to appear. (Top 3 paper out of 98 accepted, selected for presentation in the plenary session.

- Finite number of users.
- Each user can hold at most one packet in its queue.
- New packet is generated after an independent (from all other variables) exponential time with mean $1/\lambda$.
- Each packet has an independent length that is equal in distribution to a generic random variable L.
- After a collision, each participating user waits (backoffs) for an independent exponential period of time with mean $1/\nu$ and then attempts to retransmit its packet.

Visualized Scheme



We study:

- N - number of transmission attempts between 2 successful transmissions.
- T - time between 2 successful transmissions.

Power Laws in the Finite Population ALOHA with Variable Size Packets

Theorem If

$$\lim_{x\to\infty} \frac{\log \mathbb{P}[L > x]}{x} = -\mu, \quad \mu > 0,$$

then, we have

$$\lim_{n\to\infty} \frac{\log \mathbb{P}[N > n]}{\log n} = -\frac{M\mu}{(M-1)\nu}$$

and

$$\lim_{t\to\infty} \frac{\log \mathbb{P}[T > t]}{\log t} = -\frac{M\mu}{(M-1)\nu}.$$

Power Laws in the Finite Population ALOHA with Variable Size Packets

Example

4 experiments: $M = 2, 4, 10, 20$;
packet sizes $\sim$ i.i.d. exp(1);
arrival intervals and backoffs $\sim$ exp(2/3);
simulation samples $= 10^5$.
As M gets large ($M = 10, 20$), the slopes of the distributions on the log / log plot are essentially the same.

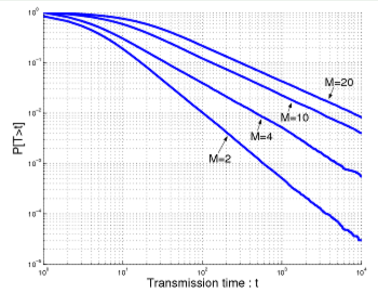


Figure: Interval distribution between successfully transmitted packets.

Implications

- the distribution tails of N and T are essentially power laws when the packet distribution $\approx e^{-\mu x}$.
- The finite population ALOHA may exhibit high variations and possible zero throughput.
  - $0 < M\mu/(M-1)\nu < 1 \Rightarrow$ zero throughput;
  - $1 < M\mu/(M-1)\nu < 2 \Rightarrow \mathbb{V}ar[T] = \infty$.
- For large M, $M\mu/(M-1) \approx \mu/\nu$ and thus, the system has zero throughput if $\nu \gtrapprox \mu$.
- High variability may even occur when $\mathbb{E}L \ll 1/\nu$.

Power Laws in Slotted ALOHA with Random Number of Users

- Power laws can be eliminated by reducing the variability of the packet sizes, however, when the number of active users M is random, we may also have power laws.

Theorem If there exists $\alpha > 0$, such that,

$$\lim_{x \to \infty} \frac{\log \mathbb{P}[M > x]}{x} = -\alpha,$$

then, we have

$$\lim_{n \to \infty} \frac{\log \mathbb{P}[N > n]}{\log n} = \lim_{t \to \infty} \frac{\log \mathbb{P}[T > t]}{\log t} = -\frac{\alpha}{\nu}.$$

More details including the proofs will be presented in the next class.

Power Laws in Slotted ALOHA with Random Number of Users

Example (Setting)

- If active users M is bounded $\Rightarrow \mathbb{P}[T > x]$ is exponentially bounded.
- However, this exponential behavior may happen for very small probabilities, while the delays of interest can fall inside the region of the distribution (main body) that behaves as the power law.
- Assume that initially $M \geq 1$ users have unit size packets ready to send and M follows geometric distribution with mean 3.
- The backoff times of colliding users are independent and geometrically distributed with mean 2.
- M has finite support $[1, K]$ where K ranges from 6 to 14 and we set the number of users to be equal to $M_K = \min(M, K)$.

Power Laws in Slotted ALOHA with Random Number of Users

Example (Simulation Results)

- The support of the main body of $\mathbb{P}[T > t]$ grows exponentially fast.
- If $K = 14$, the probabilities of interest for $\mathbb{P}[T > t]$ are bigger than $1/500$, then the result of this experiment is basically the same as for $K = \infty$.

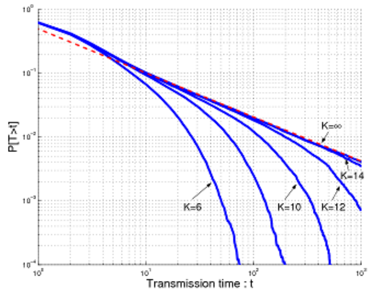


Figure: Stretched support of the power law main body when the number of users is $\min(M, K)$.