Opportunistic User Scheduling and Antenna Selection in the Downlink of Multiuser MISO Systems

Inaki Berenguer*, Ian J. Wassell*, and Xiaodong Wang[†]

 * University of Cambridge, Cambridge, UK {ib226, ijw24}@cam.ac.uk
 † Columbia University, New York, NY

wangx@ee.columbia.edu

Abstract— We consider the downlink of multiuser multipleinput single-output (MISO) wireless systems, where the base station is equipped with multiple antennas and each mobile user is constrained to a single antenna. In particular, we consider linear precoded systems such that the single antenna receivers do not have to estimate the channel, but only scale and quantize the received data. In this scenario, we propose low complexity opportunistic user scheduling and antenna selection algorithms. The highly complex optimal scheduling and antenna selection algorithms are first derived, and then, low complexity greedy optimization algorithms obtain near optimal performance.

I. INTRODUCTION

We consider a multiuser MISO wireless system consisting of a single base station and K mobile units scattered over the service area. We assume that the base station is equipped with multiple antennas and each receiver is constrained to a single antenna. Precoding schemes for broadcast channels effectively transfer the signal processing for interference suppression from the mobile receiver to the base station transmitter. This approach is feasible if the base station can estimate the downlink channels of all users (e.g., in systems employing time division duplexing (TDD) where the uplink and downlink channels are reciprocal). Various practical techniques (linear [1] and non-linear [2]) have been proposed in order to approach the downlink capacity [3]. We consider linear precoding in which the transmit signal is linearly precompensated such that the single antenna receivers simply quantize the received signal to the original symbol constellation, which translates to a reduction in power consumption, the number of training symbols needed for channel estimation, and a decrease in the cost of the terminals. One disadvantage of linear precoding in MISO systems is that to obtain reasonable performance, the number of antennas needs to be larger than the number of users served simultaneously [4], which increases the cost of the base station owing to the expensive RF blocks attached to each of the antennas. One solution to reducing the cost while maintaining the performance advantage of using more antennas than users is to use antenna selection [5]. The idea behind antenna selection is to use a limited number of the expensive RF chains and to adaptively switch them to the best subset among a larger number of available antenna elements, since antenna elements are in general inexpensive. The subset selection is performed according to a specific optimization criterion given a particular channel realization. In this paper we

propose low complexity antenna subset selection algorithms to the downlink of multiuser MISO systems.

Other advantages of precoding in multiuser MISO systems is that channel state information (CSI) at the transmitter can facilitate efficient power control and user scheduling [6] implemented jointly with linear precoding to increase the system throughput. Straightforward implementation of user subset selection in any opportunistic scheduling algorithm suffers from high computational complexity. In the second part of the paper, based on our system model we consider low complexity user selection algorithm to reduce the complexity of the scheduling solutions. In general, user subset selection can be seen as an antenna selection problem where the number of final antennas to be selected in not fixed a priori.



Fig. 1. Downlink multiuser MISO system with antenna selection.

The remainder of this paper is organized as follows. In Section II the downlink multiuser MISO system model with precoding is presented. In Section III antenna selection algorithms are considered. In Section IV user subset selection algorithms are presented, while Section IV concludes the paper.

II. SYSTEM MODEL

In the downlink of multiuser MISO systems, different data streams are transmitted for each of the users. Consider first a system with K users equipped with one antenna each and n_T transmit antennas $(n_T \ge K)$. Assume that $\boldsymbol{b} = [b_1, ..., b_K]^T$ is the transmitted symbol vector with $E\{|b_i|^2\} = 1, i = 1, ..., K$. The base station computes the precoding matrix $\boldsymbol{M} \in \mathbb{C}^{n_T \times K}$ with the knowledge of the CSI of every user and with the constraint of the total power budget available at the transmitted P_T . Then, the $n_T \times 1$ precoded signal ready to be transmitted

is given by $\boldsymbol{x} = \boldsymbol{M}\boldsymbol{b}$. By stacking together the received signal at all the mobile units in a single vector $\boldsymbol{y} = [y_1, ..., y_K]^T$ we can write

$$y = HMb + n, \tag{1}$$

where $\boldsymbol{H} \in \mathbb{C}^{K \times n_T}$ corresponds to the flat fading channel whose element h_{ij} represents the complex gain of the channel between the *j*-th transmit antenna and the *i*-th mobile unit, and n_i is the noise at the *i*-th receiver distributed as $\mathcal{N}(0, \sigma_{n,i}^2)$.

The spatial linear precoder M optimized according to the MMSE criterion is given by $M^{(u)} = H^{\dagger}$, where $(\cdot)^{\dagger}$ denotes the pseudo-inverse [1]. Notice that the precoding matrix $M^{(u)} = H^{\dagger}$ places no explicit constraint on average transmit power and a power normalization factor is required. Assuming that the total available power at the transmitter is P_T , the scaling factor is given by $\beta^2 = P_T/||Mb||^2 =$ $P_T/tr(H^{\dagger}H^{\dagger H})$ and the precoding matrix becomes M = $\beta M^{(u)} = \beta H^{\dagger}$. Then the k-th receiver makes a decision based on $y_k = \beta b_k + n_k$. With the precoding matrix M, the received SNR is equal across the users and is given by

$$\operatorname{SNR}_{k} = \frac{\beta^{2}}{\sigma_{n}^{2}} = \frac{P_{T}}{\operatorname{tr}(\boldsymbol{H}^{\dagger}\boldsymbol{H}^{\dagger H})\sigma_{n}^{2}} = \frac{P_{T}}{\operatorname{tr}\left((\boldsymbol{H}\boldsymbol{H}^{H})^{-1}\right)\sigma_{n}^{2}}.$$
 (2)

III. ANTENNA SELECTION AT THE TRANSMITTER

Although $n_T = K$ is sufficient to implement linear precoding, it has been shown in [4] that there is an optimum ratio of antennas-to-users $(n_T/K > 1)$ such that linear precoding can achieve around 80% of the sum capacity of the downlink channel computed at the same ratio. At other ratios the difference between the capacity with linear precoding and the downlink capacity can become much more pronounced. In particular, when $K = n_T$, the sum rate capacity of the linearly precoded system does not increase linearly with n_T (or K), while the capacity of the downlink channel does. Similarly, when $n_T = K$, linear precoding exhibits a poor BER performance. The optimal ratio implies that the number of transmit antennas n_T needs to be not equal but larger than the number of mobile units K. However, when multiple users K want to communicate concurrently with the base station, one major concern to implement $n_T > K$ antenna systems is the high cost due to the expense of the RF chains required for each antenna. A technique to reduce the cost of the multiple antenna system while maintaining part of the capacity is the use of antenna selection [5] where $N_T > n_T$ *inexpensive* antenna elements are installed but only n_T are actually used (see Fig. 1). Now only n_T of the more expensive RF chains is necessary. The selection algorithms for a given channel realization select the best n_T transmit antennas out of the $\binom{N_T}{n_T}$ different combinations according to a certain optimization criterion. In this paper, we choose to select the n_T antennas (i.e., n_T columns in **H**) that maximize the signal to noise ratio across the users in (2). Although a combinatorial exhaustive search of the $\binom{N_T}{n_T}$ antenna subsets can find the optimal solution, the selection would become highly complex since for every new antenna subset, a $K \times K$ matrix inverse needs to be computed. In this section, motivated by the greedy algorithms in [7] we propose sub-optimal low complexity antenna selection algorithms that only show a small loss of performance. In particular, we consider a solution using decremental selection.

The decremental selection solution begins by considering that all available antennas can be used in the transmission, and at every step, an antenna is de-activated such that the decrease in the common SNR_k is as small as possible. The process is repeated with the remaining antennas until only n_T antennas are left. Recall that removing one antenna is equivalent to removing one column in H while the rest of the columns remain unchanged.

Consider first the full matrix $H \in \mathbb{C}^{K \times N_T}$, and let h_i and H_i denote the *i*th column in H, and the submatrix of H after removing the *i*th column, respectively. Therefore, in the decremental algorithm we remove the *i*-th column in H such that the submatrix left H_i minimizes the denominator in (2), i.e.,

$$i^* = \arg\min_{i=1,\dots,N_T} \operatorname{tr}\left(\left(\boldsymbol{H}_i \boldsymbol{H}_i^H\right)^{-1}\right).$$
(3)

Notice that (3) requires the inversion of N_T matrices of size $K \times K$. Here we make use of the following equality

$$\boldsymbol{H}_{i}\boldsymbol{H}_{i}^{H} = \boldsymbol{H}\boldsymbol{H}^{H} - \boldsymbol{h}_{i}\boldsymbol{h}_{i}^{H}, \qquad (4)$$

and (3) becomes

$$\mathbf{t}^* = \arg\min_{i=1,\dots,N_T} \operatorname{tr}\left(\left(\boldsymbol{H}\boldsymbol{H}^H - \boldsymbol{h}_i^H \boldsymbol{h}_i\right)^{-1}\right).$$
(5)

Denote $A = HH^{H}$. Using the matrix inversion lemma we can write

$$\left(\boldsymbol{A}-\boldsymbol{h}_{i}^{H}\boldsymbol{h}_{i}\right)^{-1}=\boldsymbol{A}^{-1}+\boldsymbol{A}^{-1}\boldsymbol{h}_{i}\left(1-\boldsymbol{h}_{i}^{H}\boldsymbol{A}^{-1}\boldsymbol{h}_{i}\right)\boldsymbol{h}_{i}^{H}\boldsymbol{A}^{-1}.$$
(6)

Then, applying tr(U + V) = tr(U) + tr(V) we can express (5) as

$$i^* = \min_{i} \left\{ \operatorname{tr} \left(\boldsymbol{A}^{-1} \boldsymbol{h}_i \left(1 - \boldsymbol{h}_i^H \boldsymbol{A}^{-1} \boldsymbol{h}_i \right) \boldsymbol{h}_i^H \boldsymbol{A}^{-1} \right) \right\}.$$
(7)

Notice that now for the N_T possible ways of removing a single antenna, only one matrix inverse has to be computed, $A^{-1} = (HH^{H})^{-1}$. Next assume that after removing one antenna, the number of antennas is still excessive. Then, a second antenna needs to be removed from the remaining $N_T - 1$ columns in H_{i^*} , and the inverse of $(H_{i^*}H_{i^*}^H)^{-1}$ is required. However, this inverse has already been computed using the matrix inversion lemma when we removed the i^* th column in (6) (i.e., we do not need to explicitly compute a new matrix inverse at each step of the algorithm). Hence, we iteratively remove one column until only n_T antennas are left. The algorithm is shown in Algorithm 1. In the algorithm, ω denotes the set of antenna indices already selected. It is straightforward to prove that with $N_T = n_T + 1$, the algorithm is optimal. Note that the algorithm also provides us with the unconstrained precoding matrix $M^{(u)} = H[\omega]^{\dagger}$. Also note that the operations in the "update inverse" step are computed in the previous step.

Algorithm 1 Decremental antenna subset selection algorithm

Simulation Results: Consider a system with $N_T = 6$, $n_T = 5, K = 5$ and $\sigma_n^2 = 1$. We compare the BER obtained by the different antenna selection criteria with a system without antenna selection, i.e., $N_T = n_T = 5$ and a system that employs the N_T available transmit antennas, i.e., $N_T = n_T = 6$. The BER is approximated by BER = $Q(\sqrt{\text{SNR}}_k)$, which is constant across the users because of the precoding operation. Fig. 2 illustrates the BER averaged over 1000 different channel realizations. It is seen that antenna selection in MISO systems can bring an important performance improvement over systems without antenna selection and the low complexity algorithm is optimal. Note that the maximum Frobenius norm antenna selection criterion (i.e., select the antennas that see the best propagation channel in terms of power) is not a good approach in multiuser MISO systems. Even with only one extra antenna element, the performance improvement using antenna selection is considerable. It is also seen that antenna selection achieves the same diversity as when all available antennas are used, where diversity is defined as $\gamma = -\lim_{P_T \to \infty} \frac{\log \text{BER}(P_T)}{\log P_T}$ and that the power loss is around 1dB. Therefore, antenna selection can be seen as a good alternative to boost the performance of these systems. Fig. 3 shows similar results for $N_T = 12$, $n_T = 6$ and K = 6. The suboptimal decremental selection algorithm achieves almost the same performance as optimal antenna selection.

IV. DOWNLINK USER SCHEDULING

Scheduling is a technique to increase the utilization of the wireless medium. For example, in the recently proposed multiuser opportunistic scheduling scheme [6] the schedulers opportunistically exploit the channel variations of multiple users to select the *best* set of users to transmit data to subject to fairness (e.g., maximum delay), QoS (e.g., minimum SNR), and resource constraints (e.g., maximum power available at the transmitter), with the aim of achieving a significant increase of total system throughput. In general the number of users that can be simultaneously supported by the system is small and thus, there are a large number of possible user subset selections when the number of users in the system in large. Straightforward implementation of the user subset selection by simple exhaustive enumeration suffers from high computational complexity.



Fig. 2. Bit error rate for different transmit antenna selection algorithms $(N_T = 6, n_T = 5, K = 5)$.



Fig. 3. Bit error rate for different transmit antenna selection algorithms $(N_T = 12, n_T = 6, K = 6).$

In this section, we propose user subset selection algorithms that can be straight forwardly implemented in our precoded systems. For simplicity, we assume that the satisfaction that a user receives in a system (i.e., the utility) is a binary function that takes zero value when the SNR is below a threshold γ and takes a unity value when the SNR is above the threshold.

Maximum User Allocation – Optimal Solution: Our objective is to accommodate as many users as possible such that if user k is active, $SNR_k \ge \gamma$, assuming that the base station is constrained to a maximum power budget P_T . Therefore, the QoS constraint $SNR_k \ge \gamma$ translates into the following condition on H:

$$\operatorname{tr}((\boldsymbol{H}\boldsymbol{H}^{H})^{-1}) \leq \frac{P_{T}}{\sigma_{n}^{2}\gamma}.$$
(8)

Denote U as the total number of users in the network, θ as the user subset selected, and $|\theta|$ as the number of users in θ (e.g.,

selecting the first and third users corresponds to $\theta = \{1, 3\}$ and $|\theta| = 2$). The channel matrix corresponding to the active users is H_{θ} where H_{θ} is the submatrix of H (where H has U rows) obtained from the rows indicated in θ . Let Θ be the set of all possible user subsets. Therefore, the total number of possible user subsets is $|\Theta| = \sum_{k=0}^{U} {U \choose k}$. Denote Ω as the set of feasible user selections in Θ , i.e.,

$$\Omega = \{\theta \in \Theta : \mathrm{SNR}_k \ge \gamma, \ \forall k \in \theta \} \\ = \{\theta \in \Theta : \mathrm{tr}((\boldsymbol{H}_{\theta}\boldsymbol{H}_{\theta}^H)^{-1}) \le P_T / (\sigma_n^2 \gamma) \}.$$
(9)

Then the optimization problem becomes finding $\theta \in \Omega$ such that $|\theta|$ is maximized. This is a highly complex combinatorial problem since for each possible solution in Θ , a matrix pseudoinverse needs to be computed.

Low Complexity Algorithms: Next we propose lowcomplexity algorithms that employ a greedy approach to add or remove one user at a time. One important property of the linear precoder is that adding or removing one user corresponds to adding or removing a row to the channel matrix H and the rest of the rows remain unchanged. This property will allow us to propose low-complexity selection algorithms. Note that the performance only depends on the selected users and not on the order in which the users are selected. That is, for any reordering in rows of H, the required power is equivalent. Any reordering of the rows can be expressed as H' = PHwhere P is a permutation matrix and hence $P^{-1} = P^{H}$. Therefore, tr $((PHH^{H}P^{H})^{-1}) = tr ((HH^{H})^{-1})$.

Maximum Frobenius Norm Criterion: An intuitive and classical approach in user allocation is to activate the users that see the best propagation channel. Two approaches can be taken: incremental allocation and decremental allocation. In the incremental allocation algorithm, the base station starts without selecting any users. At each step of the algorithm, it selects the user with maximum channel gain (i.e., maximum norm of the corresponding row in the MIMO matrix). Then, the algorithm checks to see if (8) holds. If it does, the corresponding user is allocated. This is repeated until no more users can be allocated, i.e., until (8) no longer holds, or $|\theta| = \min(U, n_T)$. On the other hand, the decremental algorithm starts by assuming that all $|\theta| = \min(U, n_T)$ users with best channels are active. And it removes one user at a time until (8) is satisfied. The removed user is the one with the worst channel quality, i.e., with the lowest channel gain. The main disadvantage of these approaches is that for every new user added, the matrix inverse in (8) cannot be reused.

Geometrical Criterion - Incremental Selection: We have already mentioned that users with good channel qualities (i.e., large path gains) are in general good candidates to be allocated. However, due to the precoding operation, a matrix inverse needs to be computed. Therefore, users with very large path gains but with highly correlated spatial signatures (i.e., rows in the matrix H close to parallel) can have a very undesirable effect on the power required at the transmitter. Therefore here we propose to select users based not only on the gains but also on the correlations (i.e., angles) between the respective rows of the channel matrix.

Assume that $K = |\theta|$ users have already been allocated, i.e., H_{θ} with rows $h_1, ..., h_K$. Then we propose to select a new row h_i from the (U - K) remaining ones (i.e., users not allocated yet) such that the projection onto the orthogonal complement of the already selected rows is maximum, i.e.,

$$\max_{i} \|\pi^{\perp}(\boldsymbol{h}_{i})\|, \quad i \in \{\text{non-selected users}\}, \qquad (10)$$

where $\pi^{\perp}(h_i)$ denotes the projection of h_i on span $(h_1, ..., h_K)^{\perp}$ and $(\cdot)^{\perp}$ denotes the orthogonal complement. We consider a greedy incremental approach. The algorithm starts by selecting the row with the maximum norm and at every iteration the algorithm adds the row with the largest projection onto the orthogonal complement of the subspace spanned by the rows already selected. This selection can be implemented with the help of the Gram-Schmidt (GS) procedure. At every step of the algorithm, (8) needs to be checked to see if a new user can be allocated given the total power budget P_T . For every new user added, (8) requires a matrix inverse. Next, we propose a method to compute the matrix inverse recursively.

Denote the LQ decomposition of a $K \times n_T$ matrix as H = LQ where L is $K \times K$ lower left triangular and Q has dimension $K \times n_T$ with $QQ^H = I_K$. The LQ decomposition can be obtained using the GS procedure where the row vectors in Q, i.e., $q_1, ..., q_K$ are given by the recursion

$$q_1 = h_1 / \|h_1\|$$
, and $q_i = \frac{h_i - \sum_{j=1}^{i-1} \mu_{ij} q_j}{\|h_i - \sum_{j=1}^{i-1} \mu_{ij} q_j\|}$, (11)

for i = 2, ..., K, where the GS coefficients form the lower triangular matrix L and are given by

$$\mu_{ij} = \langle \boldsymbol{h}_i, \boldsymbol{q}_j \rangle, \quad j < i, \text{ and } \mu_{ii} = \| \boldsymbol{h}_i - \sum_{j=1}^{i-1} \mu_{ij} \boldsymbol{q}_j \|.$$
 (12)

By simple inspection, we have that $[\mathbf{L}]_{ij} = \mu_{ij}$, and μ_{jj} is the value required in (10). Therefore, the LQ decomposition does not require any extra computations if we use the greedy geometrical user allocation.

Assume that one knows the LQ decomposition of H. Then, (8) can be evaluated using

$$\operatorname{tr}\left((\boldsymbol{H}\boldsymbol{H}^{H})^{-1}\right) = \operatorname{tr}\left((\boldsymbol{L}\boldsymbol{Q}\boldsymbol{Q}^{H}\boldsymbol{L}^{H})^{-1}\right) = \|\boldsymbol{L}^{-1}\|_{F}^{2}.$$
 (13)

Note that (13) can be computed recursively as follows. Assume that we have computed L_{i-1}^{-1} of size $(i-1) \times (i-1)$. Then, after selecting the new user (i.e., add one row to H), the (i-1)-th leading submatrix of L_i^{-1} is given by L_{i-1}^{-1} available from the previous iteration and the last row in L_i^{-1} is given by

$$\boldsymbol{l}_{i}^{-1} = \frac{1}{\mu_{i,i}} (\boldsymbol{e}_{i} - \sum_{j=1}^{i-1} \mu_{ij} \boldsymbol{l}_{j}^{-1}), \qquad (14)$$

which follows from the Gauss-Jordan elimination and the relationship between the GS coefficients and the triangular matrix L. Hence (13) is computed recursively as

$$\|\boldsymbol{L}_{i}^{-1}\|_{F}^{2} = \|\boldsymbol{L}_{i-1}^{-1}\|_{F}^{2} + \|\boldsymbol{l}_{i}^{-1}\|_{2}^{2}.$$
(15)

Finally the low-complexity incremental user allocation is summarized in Algorithm 2. Clearly, the complexity is dominated by the computation of all the GS coefficients in step (\diamondsuit) .

Algorithm 2 Incremental user allocation – geometrical crit.
INPUT: row vectors $oldsymbol{h}_1,,oldsymbol{h}_U$, γ , P_T , σ_n .
$ heta=\emptyset$; $P_r=0$; %start without users selected
FOR $i = 1, 2,,$
FOR EVERY $j \in \{\{1,,U\} ackslash heta\}$ DO %non-selected
$\boldsymbol{b}_{j} = \boldsymbol{h}_{j} - \sum_{n=1}^{i-1} \mu_{j,p} \boldsymbol{q}_{p} \tag{(\diamond)}$
END FOR
$k_i = rg\max_j \{oldsymbol{b}_joldsymbol{b}_j^H\}$; %max orthog. projection
$oldsymbol{q}_i = oldsymbol{b}_{k_i} / \ oldsymbol{b}_{k_i}\ $; %the new GS vector
$m{l}_i^{-1} = rac{1}{\mu_{i,i}} (m{e}_i - \sum_{t=1}^{i-1} \mu_{i,t} m{l}_t^{-1})$; %last row in $m{L}^{-1}$
$P_r = P_r + \sigma_n^2 \gamma \ oldsymbol{l}_i^{-1}\ ^2$; %power required
IF $P_r < P_T$
$ heta= heta\cup k_i$; %allocate user and continue
IF $ heta =\min(U,n_T)$ THEN BREAK; %finish
ELSE
$P_r = P_r - \sigma_n^2 \gamma \ oldsymbol{l}_i^{-1}\ ^2$;
BREAK; % finish
END IF
END FOR
OUTPUT: selected users θ , required power
P_r , submatrix $oldsymbol{H}_ heta$ and $oldsymbol{H}_ heta^{\dagger} = oldsymbol{Q}^H oldsymbol{L}^{-1}$.

Simulation Results: We first consider the average number of users that each algorithm is able to allocate versus the total available power P_T (where we define the transmit power P_T relative to the noise power at the receiver, $\sigma_n^2 = 1$). We set $\gamma = 12$ dB, we assume that the transmitter has perfect CSI of all users, and we consider U = 12 available users in the region. Fig. 4 illustrates the average number of users allocated, i.e., $|\theta|$ versus P_T by the various algorithms. It is seen that the low-complexity geometrical incremental algorithm achieves almost the optimal performance. It is seen that under this scenario, the Frobenius norm selection criterion incurs a loss of between 2-4dB. Next, we consider a hypothetical scenario in which K users need to be allocated. The K users are chosen among the U available users in the network using either optimal selection, maximum gain selection, or low-complexity geometrical selection. We look at the total power required at the transmitter P_T to obtain $\gamma = 12$ dB across the K selected users. Fig. 5 shows the results with U = 16 available users, and K = 4. It is seen that the geometrical algorithm again achieves almost the optimal performance.

V. CONCLUSION

We investigated low complexity antenna selection algorithms to improve the performance of the downlink of multiuser MISO systems. We also proposed simple user subset selection algorithms that can facilitate the implementation of any opportunistic scheduling algorithm. Simulation results have shown the effectiveness of our low complexity solutions.



Fig. 4. Average number of users allocated versus the total transmit power, with U = 12 available users in the network, and target SNR $\gamma = 12$ dB.



Fig. 5. CDF of the required total power at the transmitter to allocate the best K = 4 users with target SNR $\gamma = 12$ dB, and U = 16 available users.

REFERENCES

- M. Joham, W. Utschick, and J. Nossek, "Linear transmit processing in MIMO communications systems," *IEEE Trans. Sig. Proc.*, to be published.
- [2] C. Windpassinger, R. Fischer, T.Vencel, and J. Huber, "Precoding in multi-antenna and multi-user communications," *IEEE Trans. Wireless Commun.*, vol. 3, p. 2004, Mar. 2004.
- [3] H. Weingarten, Y. Steinberg, and S. S. (Shitz), "The capacity region of the Gaussian MIMO broadcast channel," in *Proc. 38th Conf. Inform. Sci.* and Syst., (CISS), (Princeton, NJ), Mar. 2004.
- [4] C. Peel, B. Hochwald, and L. Swindlehurst, "A vector-perturbation technique for near-capacity multi-antenna multi-user communication part 2," *IEEE Trans. Commun.*, submitted.
- [5] A. Molisch and M. Win, "MIMO systems with antenna selection," *IEEE Microwave Magazine*, vol. 5, pp. 46 56, Mar 2004.
- [6] Y. Liu and E. Khightly, "Opportunistic fair scheduling over multiple wireless channels," in *Proc. IEEE INFOCOM*, (San Francisco, CA), Mar. 2003.
- [7] A. Gorokhov, D. Gore, and A. Paulraj, "Receive antenna selection for MIMO spatial multiplexing: theory and algorithms," *IEEE Trans. Sig. Proc.*, vol. 51, pp. 2796–2807, Nov. 2003.