Syllabus: CSEE E6180

I. Probability refresher

- Probability spaces: sample space, events, probability measure.
- Conditional probability, Bayes' rule
- Random variables, distributions, densities
- Expected values, moments, inequalities (Markov, Chebyshev, BCS)
- Useful distributions:
  - geometric and exponential distributions, memoryless property
  - Binomial and Poisson distributions
  - Uniform distribution, recursive property
- Transforms, generating functions
- Limit laws: Law of large numbers, central limit theorem
- Random walks

II. Simulation

- Monte Carlo methods
- Event-driven simulations
- Fundamental algorithms and data structures: linked lists, binary trees, heaps
- Generating samples from general distributions

III. Sequences of i.i.d. random variables

- Residual lifetimes
- Bernoulli and Poisson processes: relationships between the exponential, Poisson (geometric, binomial) and the uniform laws
- Superposition and decomposition
- Poisson processes in two dimensions
- Applications:
  - Execution times on faulty machines
  - Continuous polling
  - Checkpointing problems

IV. Markov chains

- Classification

- Stationary and transient distributions
- Applications:
  - Pipelined control unit
  - Disk head motion
  - Parallel program loading and execution
  - Memory interleaving

V. Two-dimensional Markov chains

- Extension of the concepts in one dimension
- Applications:
  - Positioning two disk heads
  - Storing queues in RAM

VI. Queueing analysis

- Birth and death equations, balance equations
- Infinite server ($M/M/\infty$) queue
- $M/M/1$ queue and its variants
- $M/G/1$ queue
  - Expected-value analysis
  - Generating functions
  - PASTA property, departures see time averages
  - Little's law
- Priority queues, expected-value analysis.
- Applications: Processor sharing, feedback to lower priority queues.

VII. Markov chains with the state space [0,1].

- Theory
- Applications
  - Expected head motion in a disk system
  - Continuous polling

VIII. Markov processes in higher dimensions and applications:

- Scheduling under a resource constraint
- Dynamic storage allocation
- Ring interconnection networks

IX. Single class queueing networks

- Open Jackson networks
- Closed Jackson networks
- Central server model

X. Additional applications:

- Cache system analysis
- Bus structures: multiplexing, daisy-chaining
- Paging systems, replacement algorithms