Lecture 7: Filters & Reverb

1. Filters & EQ
2. Time delay effects
3. Reverb

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1. Filters & EQ

- **EQ** is a critical tool in audio mixing
  - boost/cut on single control
  - each instrument has its own “space”

- **Different formats**
  - Low/Mid/High, Parametric

- Graphic EQ
EQ filters

• How to get **boost + cut** from a single filter?
  ○ use **allpass**

![Diagram showing allpass filter](s03-allpass1.png)

○ then +/- to get **phase** cancellation/reinforcement

![Diagram showing phase cancellation/reinforcement](s03-allpass1.png)
Allpass Filters

- Allpass filters have flat gain: $|A(e^{j\omega})| = 1$
  - from mirror-image numerator and denominator:
    $$A(z) = \frac{z^{-m}D_m(z^{-1})}{D_m(z)}$$
  - e.g. for $D_m(z) = 1 - 0.6z^{-1}$ \(\Rightarrow\) $A(z) = \frac{-0.6 + z^{-1}}{1 - 0.6z^{-1}}$

- slope governs phase interactions, group delay
  $$\tau_g(\omega_c) = -\frac{d\theta(\omega)}{d\omega} \bigg|_{\omega=\omega_c}$$
Group Delay

- Local phase change of $H(z)$ governs effective delay of that frequency region: “group delay”

$$
\tau_g(\omega_c) = -\frac{d\theta(\omega)}{d\omega} \bigg|_{\omega=\omega_c}
$$
Parametric EQ

- 2nd order Allpass $\rightarrow$ slope & place $\theta(\omega) = \pi$

- angle $\rightarrow$ frequency
- radius $\rightarrow$ slope $\rightarrow$ bandwidth

- Same structure for EQ

\[ A(z) + g = -1 \ldots 1 \]
Time-Varying Filters

- Classic Wah-Wah?

- Iterated Filters...
2. Time Delays

- Delays correspond to sound propagation
  - $340 \, \text{m/s} \approx 1 \, \text{foot / ms}$

- Delays are a simple kind of filter
  - can analyze from Fourier perspective...

\[
x[n] \quad \text{delay } \tau \quad y[n]
\]

\[
h[n] = \delta[n - \tau] \quad \Rightarrow H(e^{j\omega}) = e^{-j\omega\tau}
\]
Fractional Delays

• For short delays, one sample quantization may be too coarse
  - 1 sample @ 44.1 kHz = 22.7 µs

• **Fractional delay** can be recovered from Fourier domain

\[ e^{-j \omega \tau} \Leftrightarrow \frac{\sin \pi (n - \tau)}{\pi (n - \tau)} = \text{sinc}(n - \tau) \]

- truncated FIR implementation
Comb Filters

- Delay added to direct path causes “comb”
  - from phase interactions

- Range of perceptual effects
  - < 10 ms - phasing (spectral structure)
  - 20-100 ms - chorus/doubling
  - > 100 ms - echo
IIR Comb Filters

- Feedback delay spreads out more in time

- Poles can give unbounded gain
Time Varying Delay

- Periodic variation over large range
  $\Rightarrow$ Pitch modulation
  - analogous to Doppler shift

- Random (but smooth) shift over short delay
  $\Rightarrow$ Chorus
  - pattern of cancellation “notches” like detuned voices
3. Reverberation

- Received sound is **direct path + reflections**
  - delayed relative to direct path
  - different at each ear
  - Direct-to-Reverberant...

- **Source → Ear is ~ LTI**
  - can use impulse response, convolution
Early Echoes & Late Reverb

- Reflected paths are like “virtual sources”

- First part of reverberant IR is sparse

- Reflections quickly build up & merge
  - Later part of reverb is like decaying noise
Nested Allpass

- **Allpass** efficiently creates decaying response
  - multiple, combined filters for **complex** patterns

\[ H(z) = \frac{z^{-k} - g}{1 - g \cdot z^{-k}} \]

\[ h[n] = \begin{cases} 1 & \text{if } k = n \mod 3 \\ 0 & \text{otherwise} \end{cases} \]

Synthetic Reverb

**FIR Comb**

**iIR Comb**

**Real Part**

**Imaginary Part**
Feedback Delay Network

- **Matrix** of feedbacks gives even more complex patterns
  - “Unitary” matrix ensures decay
Summary

- **Filters:**
  - EQ used to balance mixes
  - Varying filters gives effects e.g. Wah-wah

- **Delays**
  - Wide range of effects: phasing ... echo
  - Fractional delays

- **Reverb**
  - Just a complex pattern of echoes
  - Discrete early echoes → reverberant tail