6.1 Show that for a causal sequence \( x[n] \) defined for \( n \geq 0 \) and with a \( z \)-transform \( X(z) \),
\[
x[0] = \lim_{z \to \infty} X(z).
\]
The above result is known as the initial value theorem.

6.5 Consider the following sequences:
(i) \( x_1[n] = (0.3)^n \mu[n + 1] \),
(ii) \( x_2[n] = (0.7)^n \mu[n - 1] \),
(iii) \( x_3[n] = (0.4)^n \mu[n - 5] \),
(iv) \( x_4[n] = (-0.4)^n \mu[-n - 2] \).
(a) Determine the ROCs of the \( z \)-transform of each of the above sequences.
(b) From the ROCs determined in Part (a), determine the ROCs of the following sequences:
(i) \( y_1[n] = x_1[n] + x_2[n] \),
(ii) \( y_2[n] = x_1[n] + x_3[n] \),
(iii) \( y_3[n] = x_1[n] + x_4[n] \),
(iv) \( y_4[n] = x_2[n] + x_3[n] \),
(v) \( y_5[n] = x_2[n] + x_4[n] \),
(vi) \( y_6[n] = x_3[n] + x_4[n] \).

6.7 Determine the \( z \)-transform of each of the following sequences and their respective ROCs. Assume \(|\beta| > |\alpha| > 0\).
Show their pole-zero plots and indicate clearly the ROC in these plots.
(a) \( x_1[n] = (\alpha^n + \beta^n) \mu[n + 2] \),
(b) \( x_2[n] = \alpha^n \mu[-n - 2] + \beta^n \mu[n - 1] \),
(c) \( x_3[n] = \alpha^n \mu[n + 1] + \beta^n \mu[-n - 2] \).

6.20 Each one of following \( z \)-transforms
\[
X_a(z) = \frac{3z}{z^2 + 0.3z - 0.18}, \quad X_b(z) = \frac{3z^2 + 0.1z + 0.87}{(z + 0.6)(z - 0.3)^2}
\]
has three ROCs. Evaluate their respective inverse \( z \)-transforms corresponding to each ROC.

M 6.1 Using Program 6.1, determine the factored form of the following \( z \)-transforms:
(a) \( G_1(z) = \frac{2z^4 - 5z^3 + 13.48z^2 - 7.78z + 9}{4z^4 + 7.2z^3 + 20z^2 - 0.8z + 8} \),
(b) \( G_2(z) = \frac{5z^4 + 3.5z^3 + 21.5z^2 - 4.6z + 18}{5z^4 + 15.5z^3 + 31.7z^2 + 22.52z + 4.8} \),
and show their pole-zero plots. Determine all possible ROCs of each of the above \( z \)-transforms, and describe the type of their inverse \( z \)-transforms (left-sided, right-sided, two-sided sequences) associated with each of the ROCs.