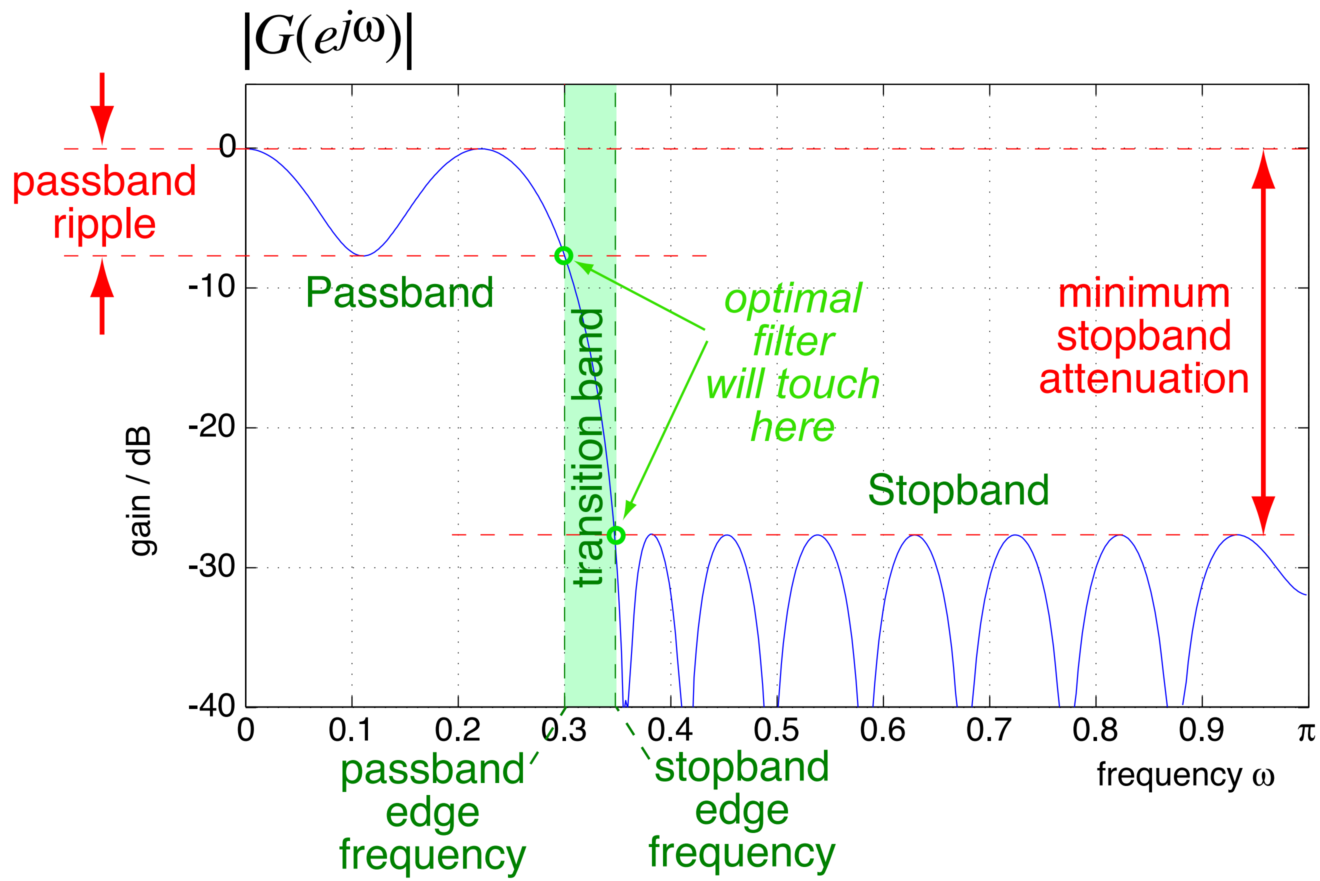
ELEN E4810: Digital Signal Processing

Review Session

1. Filter design
2. Allpass & Minimum phase
3. IIR filter design
4. FIR filter design
5. Implementations
6. FFT

Filter Design

- Filters select frequency regions
- Performance Margins



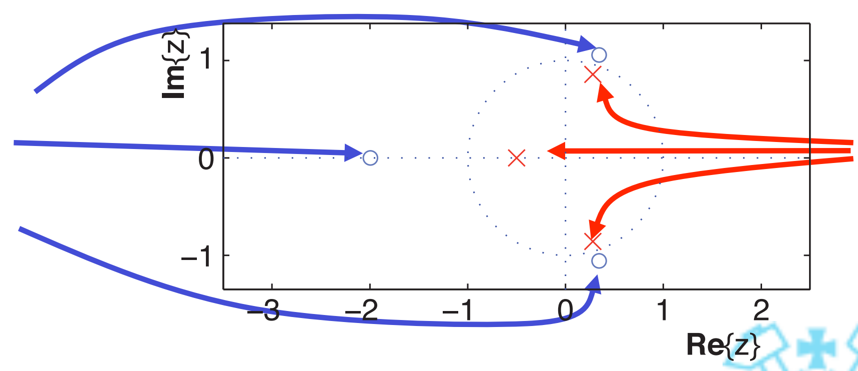
Allpass Filters

- Constant gain, variable phase
- Mirror-image polynomial

$$A_M(z) = \pm \frac{d_M + d_{M-1}z^{-1} + \ldots + d_1 z^{-(M-1)} + z^{-M}}{1 + d_1 z^{-1} + \ldots + d_{M-1} z^{-(M-1)} + d_M z^{-M}}$$

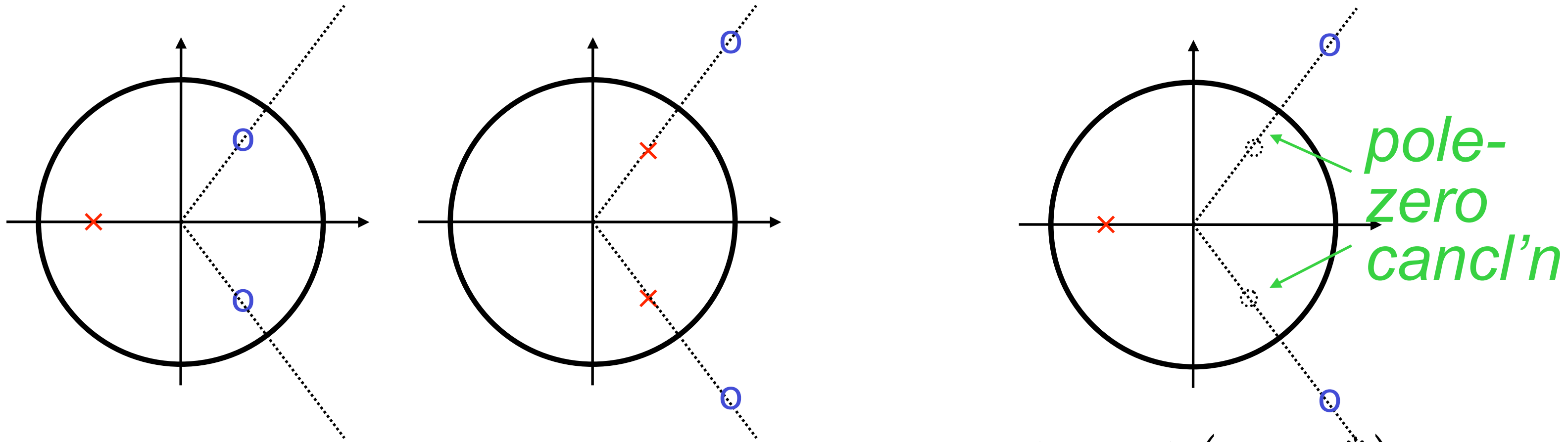
$$= \pm z^{-M} \frac{D_M\left(z^{-1}\right)}{D_M(z)}$$

- Reciprocal poles/zeros



Minimum & Maximum Phase

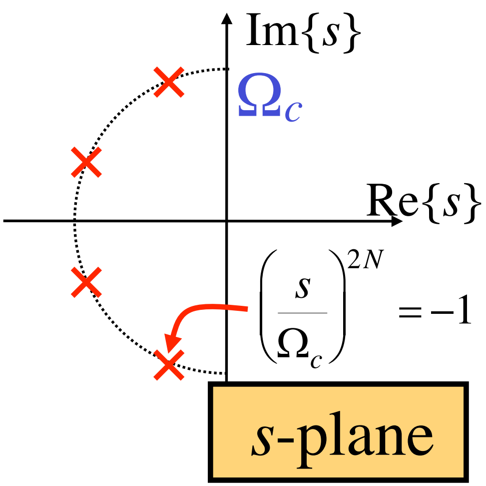
- Min. phase + Allpass = Max. phase

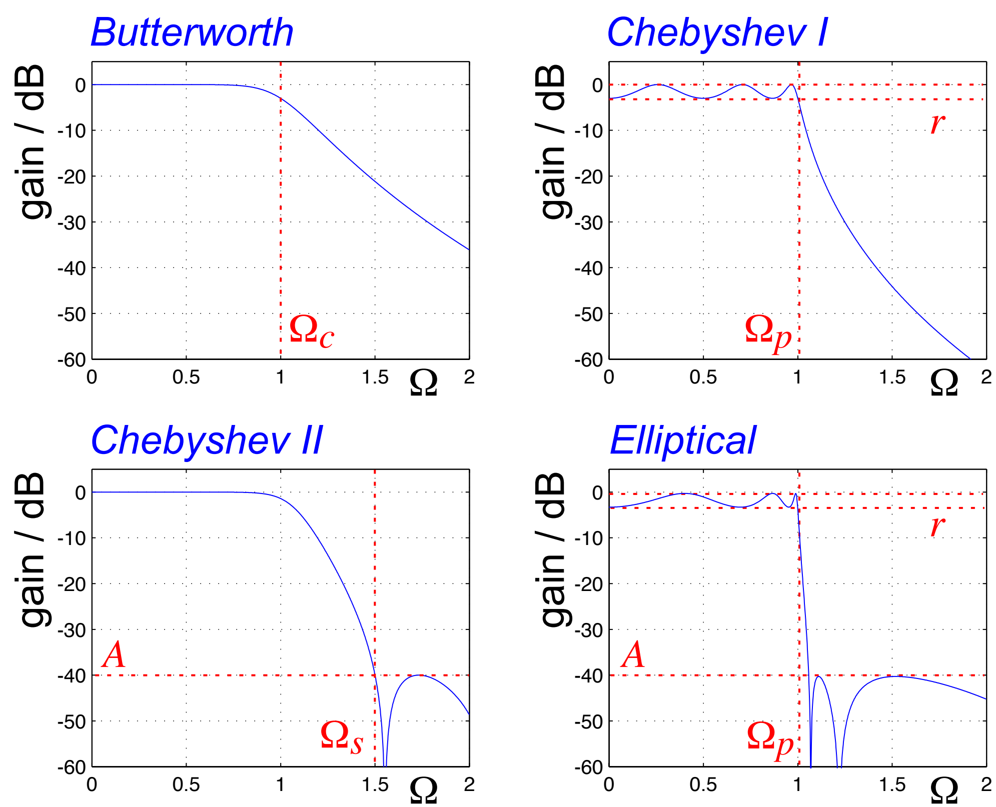


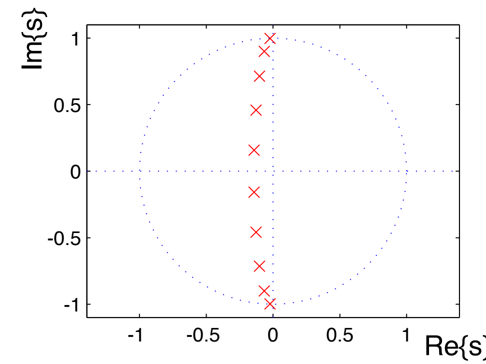
$$\frac{(z-\zeta)(z-\zeta^*)}{z-\lambda} \times \frac{\left(z-\frac{1}{\zeta}\right)\left(z-\frac{1}{\zeta}^*\right)}{(z-\zeta)(z-\zeta^*)} = \frac{\left(z-\frac{1}{\zeta}\right)\left(z-\frac{1}{\zeta}^*\right)}{z-\lambda}$$

Analog Filter Types

- {flat | ripple} x {passband | stopband}



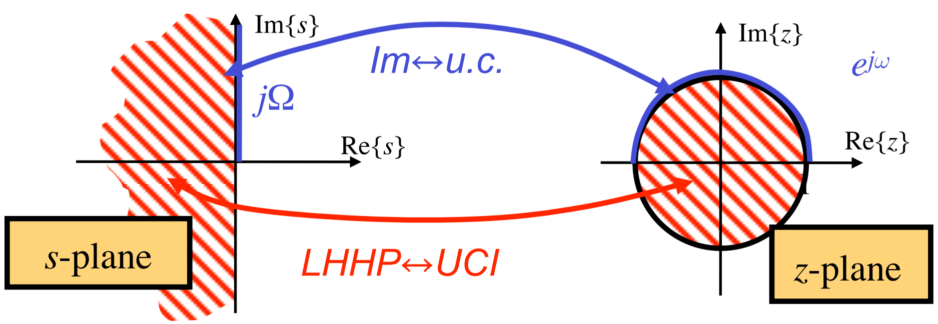


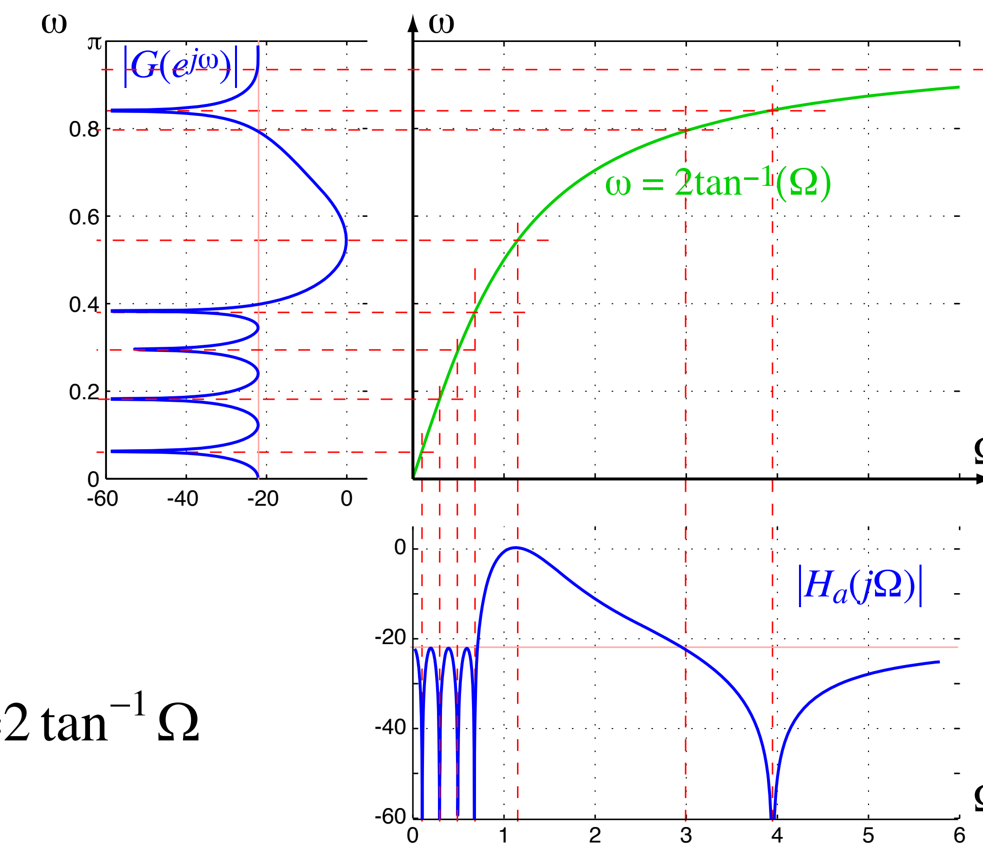


Bilinear Transform

$$s = \frac{1 - z^{-1}}{1 + z^{-1}} = \frac{z - 1}{z + 1}$$

- Analog IIR to Discrete-time IIR



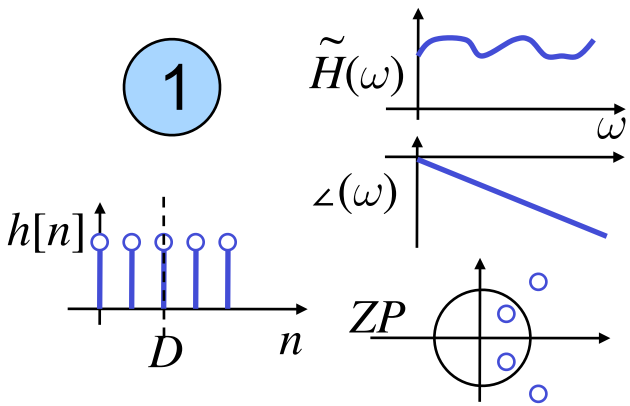


$$G\left(e^{j\omega}\right) = H_a(j\Omega)\Big|_{\omega = 2\tan^{-1}\Omega}$$

- "Pre-warp" to design

FIR Filters

- Linear Phase FIR filters, e.g. $h[n] = h[N-n]$



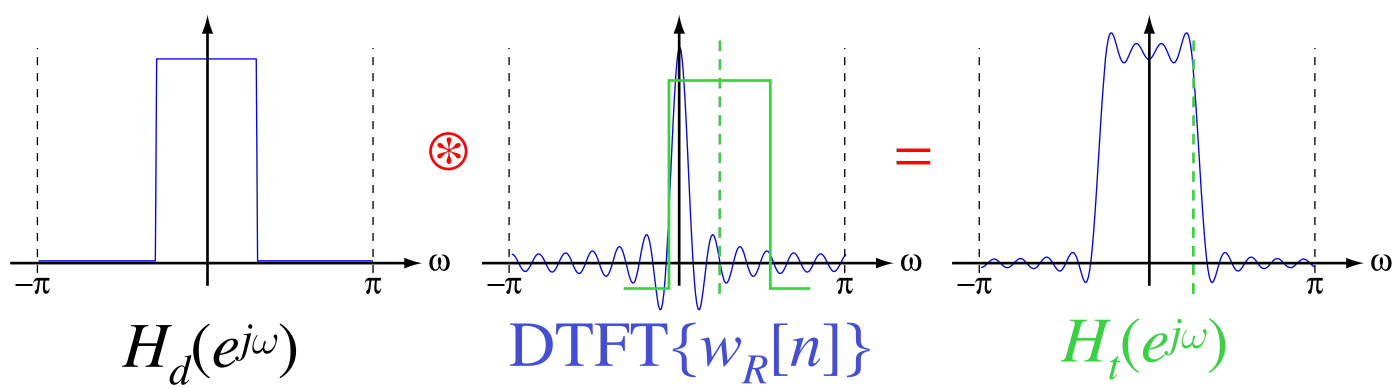
$$H(e^{j\omega}) = \sum_{n=0}^{N} h[n]e^{-j\omega n}$$

$$= e^{-j\omega\frac{N}{2}}\left(h[\tfrac{N}{2}] + 2\sum_{n=1}^{N/2} h[\tfrac{N}{2}-n]\cos\omega n\right)$$

linear phase

$D = -\theta(\omega)/\omega = N/2$

- Windowed ideal responses



Parks-McClellan FIR design

- Gradient descent to find best FIR filter
- At least M+2 alternating extrema (order = 2M, length = order + 1)

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>> h=firpm(10, [0 0.4 0.6 1],
               [1 1 0 0], [1 2]);
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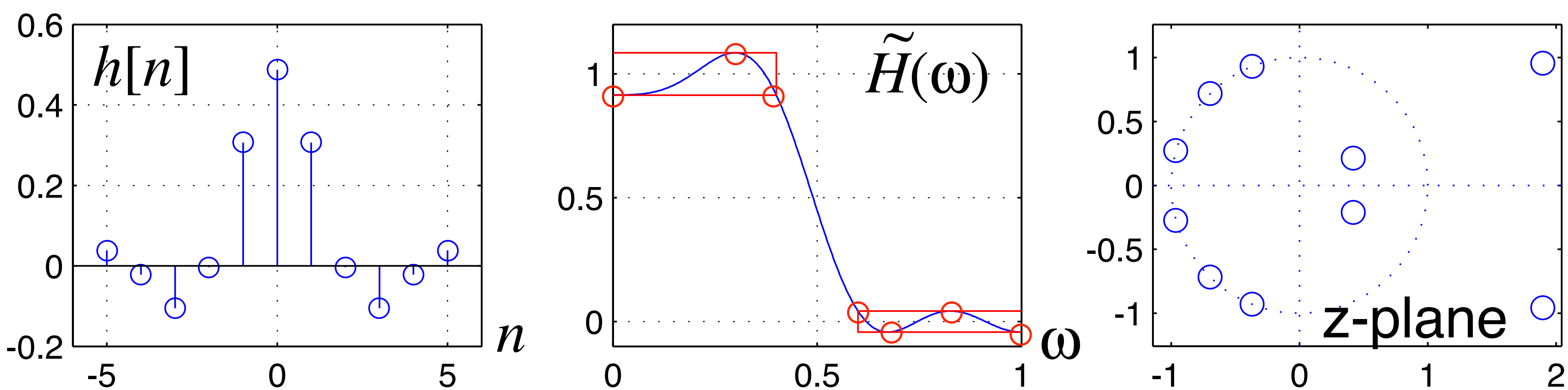
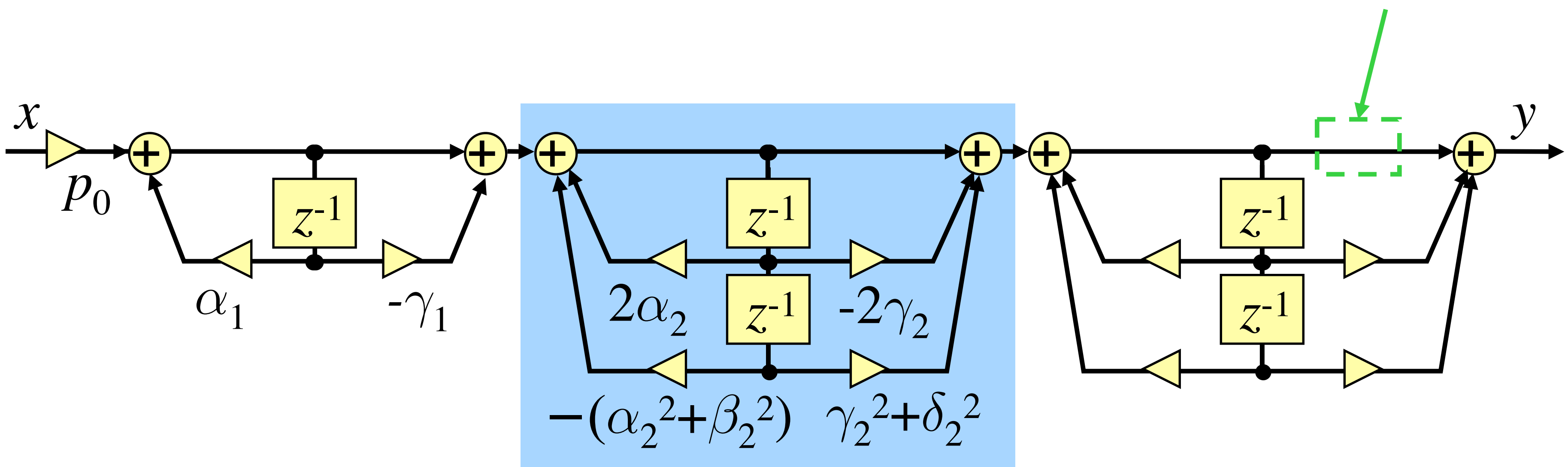


Implementations

- Polynomial indicates implementation
- Decompose into common blocks (second order sections)



FFT

$$X[k] = \sum_{n=0}^{N-1} x[n] \cdot W_N^{nk}$$

$$= \sum_{m=0}^{\frac{N}{2}-1} x[2m] \cdot W_{\frac{N}{2}}^{mk} + W_N^k \sum_{m=0}^{\frac{N}{2}-1} x[2m+1] \cdot W_{\frac{N}{2}}^{mk}$$

$X_0[\langle k \rangle_{N/2}]$ — *N/2 pt DFT of x for* ***even*** *n*

$X_1[\langle k \rangle_{N/2}]$ — *N/2 pt DFT of x for* ***odd*** *n*