ELEN E4810: Digital Signal Processing

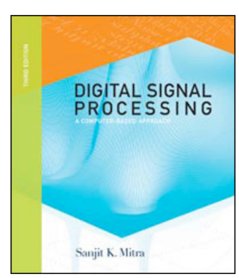
# Topic 1: Introduction

1. Course overview
2. Digital Signal Processing
3. Basic operations & block diagrams
4. Classes of sequences



# 1. Course overview

- **Digital signal processing**:
  Modifying signals with computers
- Web site:
  http://www.ee.columbia.edu/~dpwe/e4810/
- Book:
  Mitra "Digital Signal Processing"
  (3rd ed., 2005)




- Instructor: dpwe@ee.columbia.edu

## Grading structure

- Homeworks: 20%
  - Mainly from Mitra
  - Tuesday-tuesday schedule
  - Collaborate, don't copy
- Midterm: 20%
  - One session
- Final exam: 30%
- Project: 30%



## Course project

- Goal: hands-on experience with DSP
- Practical implementation
- Work in pairs or alone
- Brief report, optional presentation
- Recommend MATLAB
- Ideas on website

## Example past projects

on web site

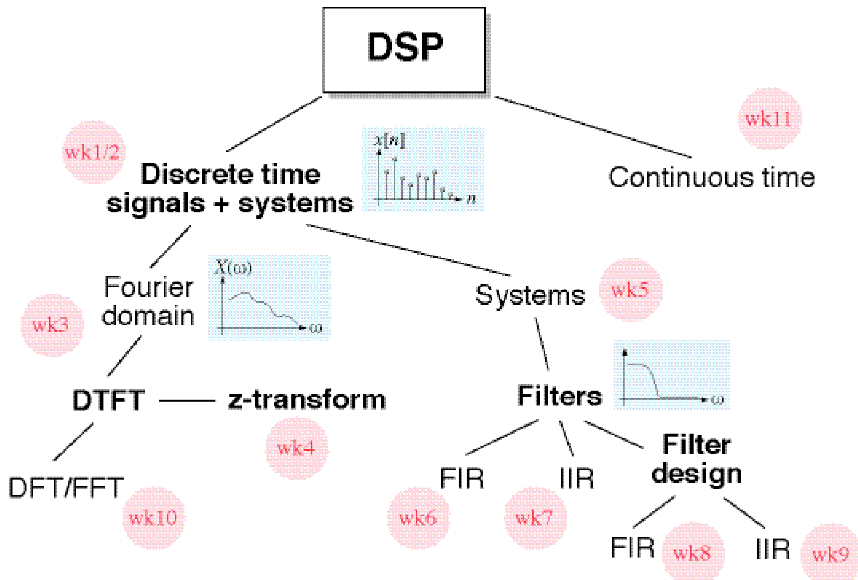
- Solo Singing Detection
- Guitar Chord Classifier
- Speech/Music Discrimination
- DTMF decoder
- Reverb algorithms
- Compression algorithms



## MATLAB

- Interactive system for numerical computation
- Extensive signal processing library
- Focus on algorithm, not implementation
- Access:
  - Engineering Terrace 251 computer lab
  - Student Version (need Sig. Proc. toolbox)
  - ILAB, 1235 Mudd (email TA)

# Course at a glance






# 2. Digital Signal Processing

- Signals:
  Information-bearing function
- E.g. sound: air pressure variation at a point as a function of time $p(t)$
- Dimensionality:
  Sound: 1-Dimension
  Greyscale image $i(x,y)$ : 2-D
  Video: 3 x 3-D: $\{r(x,y,t)\ g(x,y,t)\ b(x,y,t)\}$

## Example signals

- Noise - all domains
- Spread-spectrum phone - radio
- ECG - biological
- Music
- Image/video - compression
- ….

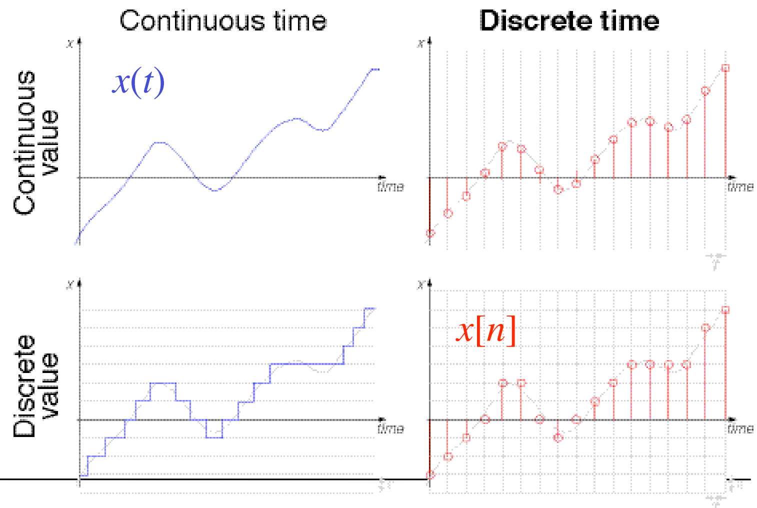


## Signal processing

- Modify a signal to extract/enhance/ rearrange the information
- Origin in analog electronics e.g. radar
- Examples…
  - Noise reduction
  - Data compression
  - Representation for recognition/ classification…

# Digital Signal Processing

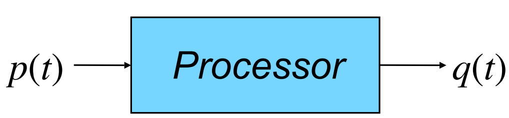
- DSP = signal processing on a computer
- Two effects: discrete-time, discrete level



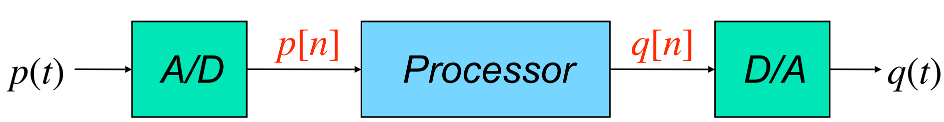



# DSP vs. analog SP

- Conventional signal processing:




- Digital SP system:



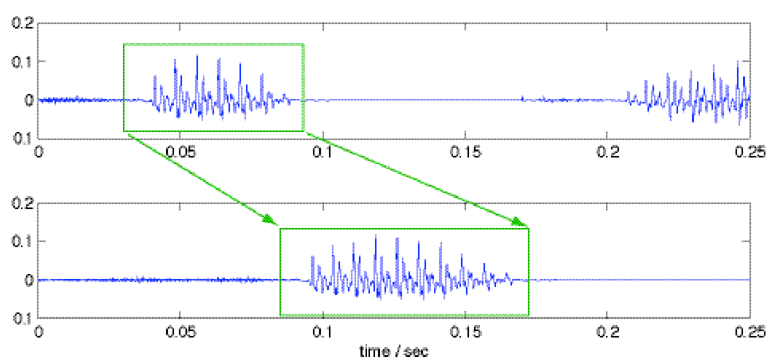
# Digital vs. analog

- Pros
  - Noise performance - quantized signal
  - Use a general computer - flexibility, upgrde
  - Stability/duplicability
  - Novelty
- Cons
  - Limitations of A/D & D/A
  - Baseline complexity / power consumption



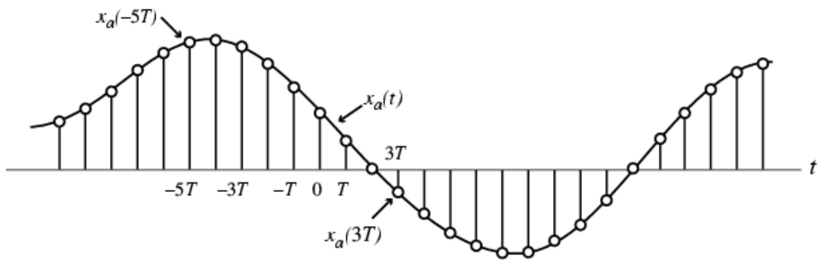
# DSP example

- Speech time-scale modification:
  extend duration without altering pitch



# 3. Operations on signals

- Discrete time signal often obtained by sampling a continuous-time signal




- Sequence $\{x[n]\} = x_a(nT)$, $n = \ldots -1, 0, 1, 2 \ldots$
- $T$= samp. period; $1/T$= samp. frequency



# Sequences

- Can write a sequence by listing values:

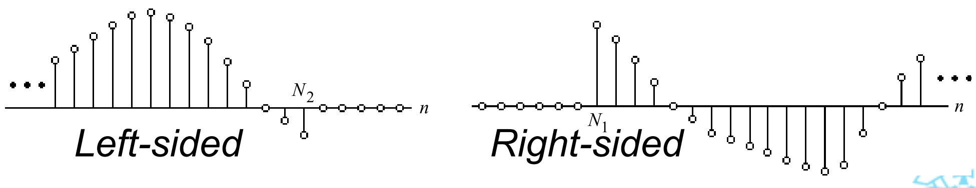
$$\{x[n]\} = \{\ldots, -0.2, 2.2, 1.1, 0.2, -3.7, 2.9, \ldots\}$$

!

  - Arrow indicates where $n$=0
  - Thus, $x[-1] = -0.2,\ x[0] = 2.2,\ x[1] = 1.1,$

# Left- and right-sided

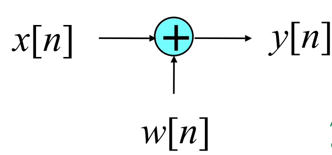
- $x[n]$ may be defined only for certain $n$:
  - $N_1 \le n \le N_2$: Finite length (length = …)
  - $N_1 \le n$: Right-sided (Causal if $N_1 \ge 0$)
  - $n \le N_2$: Left-sided (Anticausal)
- Can always extend with zero-padding





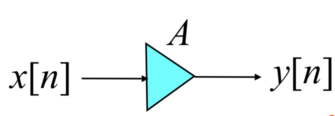

# Operations on sequences

- **Addition** operation:
  - Adder




$$y[n] = x[n] + w[n]$$

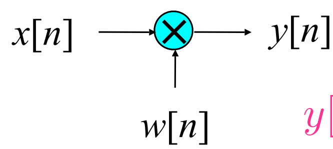
- **Multiplication** operation
  - Multiplier




$$y[n] = A \times x[n]$$

# More operations

- **Product** (**modulation**) operation:
  - Modulator



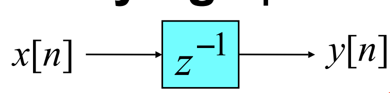

$$y[n] = x[n] \times w[n]$$

- E.g. Windowing: multiplying an infinite-length sequence by a finite-length window sequence to extract a region



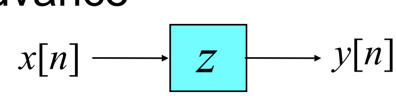
# Time shifting

- **Time-shifting** operation: $y[n] = x[n-N]$ where *N* is an integer
- If *N* > 0, it is **delaying** operation
  - Unit delay




$$y[n] = x[n-1]$$

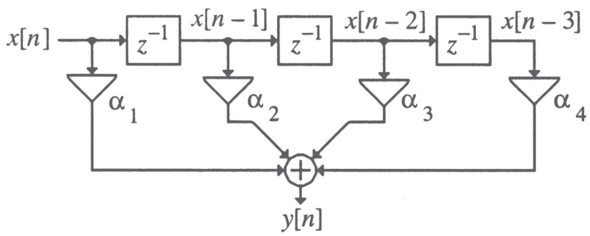
- If *N* < 0, it is an **advance** operation
  - Unit advance




$$y[n] = x[n+1]$$

# Combination of basic operations

- Example




$$y[n] = \alpha_1 x[n] \; + \; \alpha_2 x[n-1] \; + \; \alpha_3 x[n-2] \; + \; \alpha_4 x[n-3]$$



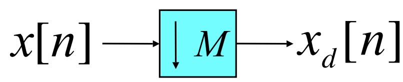
# Up- and down-sampling

- Certain operations change the effective sampling rate of sequences by adding or removing samples
- Up-sampling = adding more samples = interpolation
- Down-sampling = discarding samples = decimation

# Down-sampling

- In **down-sampling** by an integer factor $M > 1$, every $M$-th sample of the input sequence is kept and $M$ - 1 in-between samples are removed:

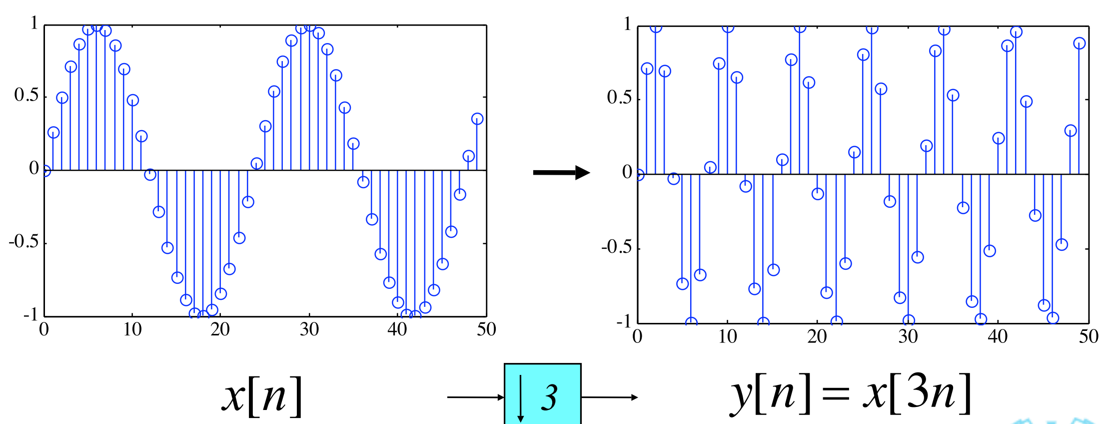
$$x_d[n] = x[nM]$$






# Down-sampling

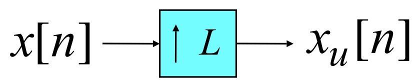
- An example of down-sampling



# Up-sampling

- Up-sampling is the converse of down-sampling: $L$-1 zero values are inserted between each pair of original values.

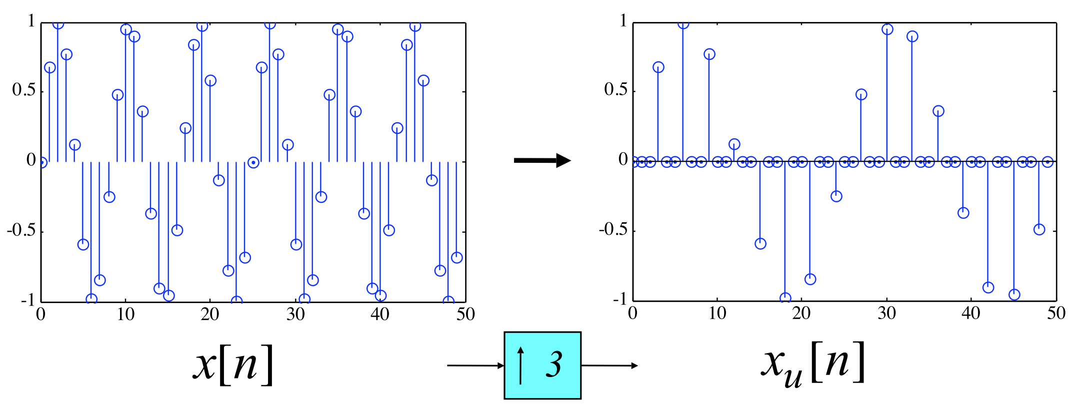
$$x_u[n] = \begin{cases} x[n/L] & n = 0, \pm L, 2L, \ldots \\ 0 & \text{otherwise} \end{cases}$$






# Up-sampling

- An example of up-sampling




***not** inverse of downsampling!*

## Complex numbers

- .. a mathematical convenience that lead to simple expressions
- A second "imaginary" dimension (j≡√-1) is added to all values.
- **Rectangular** form: $x = x_{re} + j \cdot x_{im}$

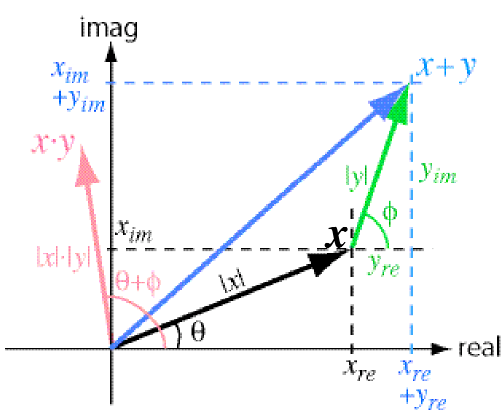
  where *magnitude* $|x| = \sqrt{(x_{re}^2 + x_{im}^2)}$

  and *phase* " $= \tan^{-1}(x_{im}/x_{re})$
- **Polar** form: $x = |x|$ ej" $= |x|$cos" $+ j \cdot |x|$sin"

  $( e^{j\theta} = \cos\theta + j\sin\theta )$



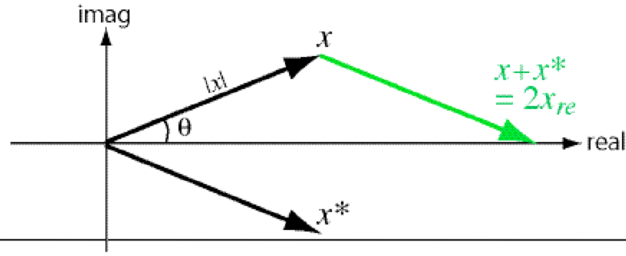
## Complex math




- When **adding**, real and imaginary parts add: $(a+jb) + (c+jd)$
  $= (a+c) + j(b+d)$
- When **multiplying**, magnitudes multiply and phases add:
  $r$ej" $\cdot s$ej# $= rs$ej(" +#)
- Phases modulo 2$

# Complex conjugate

- Flips imaginary part / negates phase: conjugate $x^* = x_{re} - j \cdot x_{im} = |x| \, e^{j(-\theta)}$
- Useful in resolving to real quantities:
  $x + x^* = x_{re} + j \cdot x_{im} + x_{re} - j \cdot x_{im} = 2x_{re}$
  $x \cdot x^* = |x| \, e^{j(\theta)} \, |x| \, e^{j(-\theta)} = |x|^2$



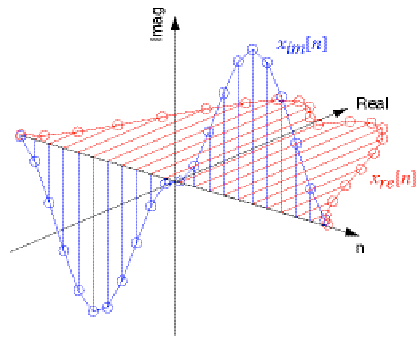



# Classes of sequences

- Useful to define broad categories…
- Finite/infinite (extent in $n$)
- Real/complex:
  $x[n] = x_{re}[n] + j \cdot x_{im}[n]$

# Classification by symmetry

- Conjugate symmetric sequence:
  $x_{cs}[n] = x_{cs}^*[-n] = x_{re}[-n] - j \cdot x_{im}[-n]$




- Conjugate antisymmetric:
  $x_{ca}[n] = -x_{ca}^*[-n] = -x_{re}[-n] + j \cdot x_{im}[-n]$



# Conjugate symmetric decomposition

- Any sequence can be expressed as conjugate symmetric (CS) / antisymmetric (CA) parts:

  $$x[n] = x_{cs}[n] + x_{ca}[n]$$

  where:

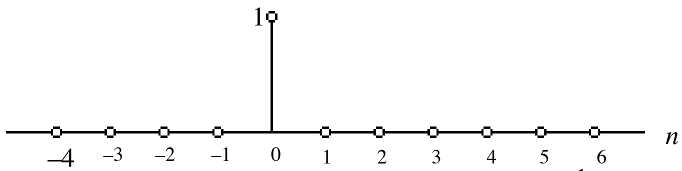
  $$x_{cs}[n] = {}^1/_2(x[n] + x^*[-n]) \quad = x_{cs}^*[-n]$$

  $$x_{ca}[n] = {}^1/_2(x[n] - x^*[-n]) \quad = -x_{ca}^*[-n]$$

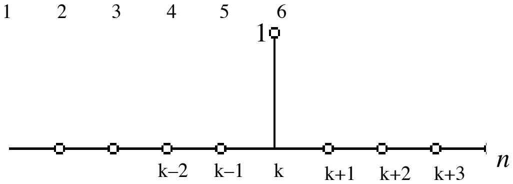
- When signals are real,
  CS % Even ($x_{re}[n] = x_{re}[-n]$), CA % Odd

# Basic sequences

- Unit sample sequence: $\delta[n] = \begin{cases} 1, & n = 0 \\ 0, & n \neq 0 \end{cases}$




- Shift in time:
  $\delta[n - k]$

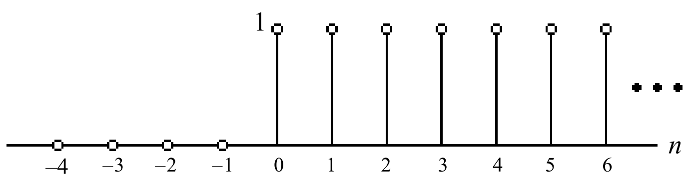



- Can express any sequence with $\delta$:
  $\{\alpha_0, \alpha_1, \alpha_2 ..\} = \alpha_0 \delta[n] + \alpha_1 \delta[n-1] + \alpha_2 \delta[n-2] ..$



# More basic sequences

- Unit step sequence: $\mu[n] = \begin{cases} 1, & n \geq 0 \\ 0, & n < 0 \end{cases}$



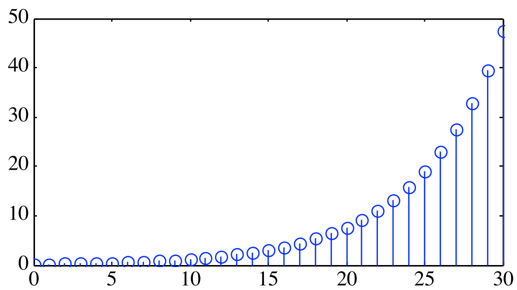

- Relate to unit sample:

$$\delta[n] = \mu[n] - \mu[n-1]$$

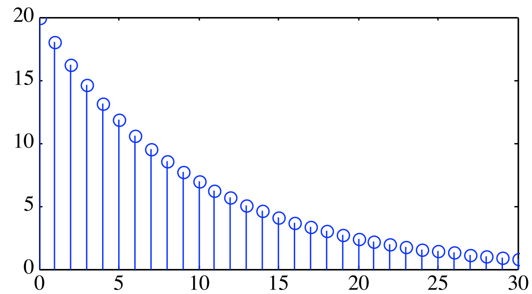
$$\mu[n] = \sum_{k=-\infty}^{n} \delta[k]$$

# Exponential sequences

- Exponential sequences= *eigenfunctions*
- General form: $x[n] = A \cdot \alpha^n$
- If $A$ and $\alpha$ are *real* (and positive):



$|\alpha| > 1$



$|\alpha| < 1$



# Complex exponentials

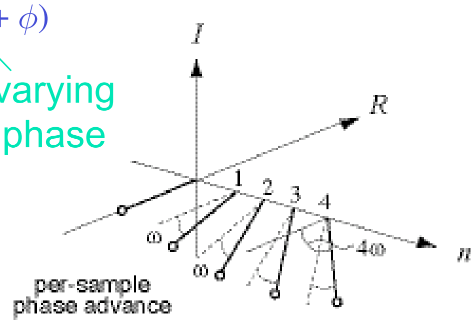
$x[n] = A \cdot \alpha^n$

- Constants $A$, $\alpha$ can be complex :

$A = |A| e^{j\phi}$ ; $\alpha = e^{(\sigma + j\omega)}$

% $x[n] = |A|\, e^{\sigma n}\, e^{j(\omega n + \phi)}$

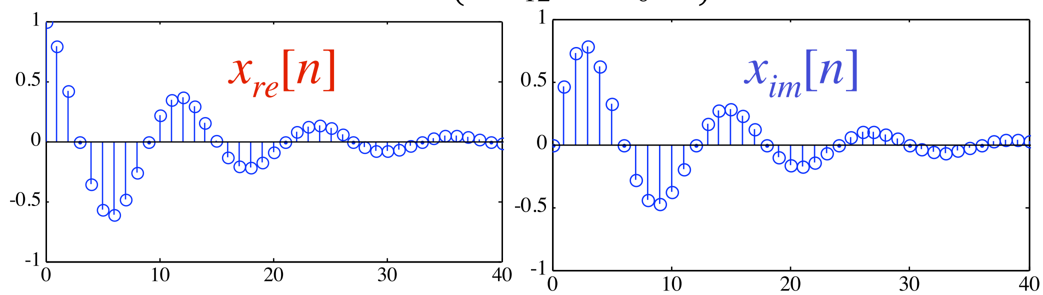
scale — varying magnitude — varying phase



# Complex exponentials

- Complex exponential sequence can 'project down' onto real & imaginary axes to give sinusoidal sequences

$$x[n] = \exp\left\{(-\tfrac{1}{12} + j\tfrac{\pi}{6})n\right\} \qquad e^{j\theta} = \cos\theta + j\sin\theta$$



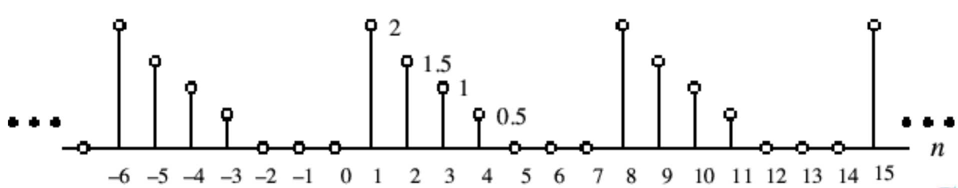

$x_{re}[n] = e^{-n/12}\cos(\pi n/6)$ $x_{im}[n] = e^{-n/12}\sin(\pi n/6)$ M



# Periodic sequences

- A sequence $\tilde{x}[n]$ satisfying $\tilde{x}[n] = \tilde{x}[n + kN]$, is called a **periodic sequence** with a **period** $N$ where $N$ is a positive integer and $k$ is any integer.

  Smallest value of $N$ satisfying $\tilde{x}[n] = \tilde{x}[n + kN]$ is called the **fundamental period**



# Periodic exponentials

- Sinusoidal sequence $A\cos(\omega_o n + \phi)$ and complex exponential sequence $B\exp(j\omega_o n)$ are periodic sequences of period $N$ **only if** $\omega_o N = 2\pi r$ with $N$ & $r$ positive integers
- Smallest value of $N$ satisfying $\omega_o N = 2\pi r$ is the **fundamental period** of the sequence
- $r = 1$ % one sinusoid cycle per $N$ samples
  $r > 1$ % $r$ cycles per $N$ samples

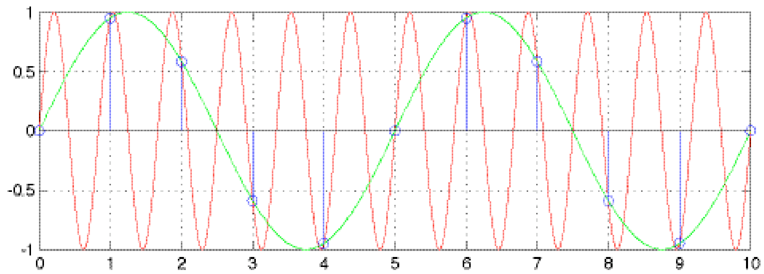


# Symmetry of periodic sequences

- An $N$-point finite-length sequence $x_f[n]$ defines a periodic sequence:
  $x[n] = x_f[<n>_N]$ "$n$ modulo $N$"
- Symmetry of $x_f[n]$ is not defined because $x_f[n]$ is undefined for $n < 0$
- Define Periodic Conjugate Symmetric:
  $$x_{pcs}[n] = {}^1/_2(x[n] + x^*[<-n>_N])$$
  $$= {}^1/_2(x_f[n] + x_f^*[N-n]) \qquad 1 \le n < N$$

## Sampling sinusoids

- Sampling a sinusoid is *ambiguous*:




$x_1[n] = \sin(\omega_0 n)$

$x_2[n] = \sin((\omega_0 + 2\pi r)n) = \sin(\omega_0 n) = x_1[n]$



## Aliasing

- E.g. for $\cos(\omega n),\ \omega = 2\pi r \pm \omega_0$

  all $r$ appear the same after sampling
- We say that a larger $\omega$ appears aliased to a lower frequency
- **Principal value** for discrete-time frequency: $0 \le \omega_0 \le \pi$ ( i.e. less than one-half cycle per sample)