

Wednesday 2011-10-24 10:10-11:25 (75 min)

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This test consists of only one question.

You have one and one quarter hours (75 minutes) to complete the test.

This test is open-book: you are permitted to refer to your notes and textbooks during the test.

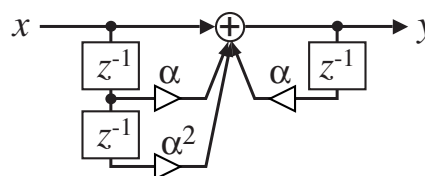
You may use a calculator for numerical work, but not for graphing. You cannot use Matlab or similar.

You must show all your workings to get credit for an answer.

You are on your honor to present work that is entirely your own.

1. (a) For the system described by the block diagram below, find:

- i. the difference equation
- ii. the z -transform, $H(z)$
- iii. the magnitude of z for all points where $|H(z)| = 0$ or $|H(z)| = \frac{1}{0}$
- iv. the pole-zero diagram



- (b) What is the relationship between the roots of the system and the value of α ?
- (c) On a single graph, sketch the magnitude response for a range of values of α between 0.5 and 1. Include at least three curves, to show how the magnitude response changes with α .
- (d) Rewrite $H(z)$ in terms of $u = z/\alpha$. Can you draw a modified block diagram that reflects this substitution (i.e., composed of αz^{-1} blocks)?
- (e) What is the impulse response, $h[n]$, of the system?
- (f) What is the 4-point DFT, $H[k] = H(e^{j\omega})|_{\omega=2\pi k/N}$ for $k = 0 \dots 3$ and $N = 4$, of this impulse response when $\alpha = 0.5$? (Note that the impulse response has more than 4 points.)