EE E4703 Wireless Communications Spring
Professor Diament
Solution to Homework Assignment #5

Problem 6.1

Given $f_m = \frac{5}{2\pi} = 0.796Hz$, $\beta_f = 10 \Rightarrow$ bandwidth of the FM signal $B_t = 2(\beta_f + 1)f_m \Rightarrow B_T = 17.5Hz$.

For $f_c = \frac{\omega_c}{2\pi} = 796.18Hz$, the upper sideband frequency is f_c to $f_c + \frac{B_T}{2} =$ 796.18 to $804.93Hz$.

The lower sideband frequency is $f_c - \frac{B_T}{2}$ to $f_c =$ 787.43 to $796.18Hz$.

Problem 6.2

For $m(t) = \sin(1000\pi t), \Delta F = 1\text{KHz}$, $A_m = 2\text{V}$, $f_m = \frac{1000\pi}{2\pi} = 500\text{Hz}$, we have $k_f = \frac{\Delta f}{A_m} = \frac{1000}{2} = 500\text{Hz/V}$.

Thus, for $A_m = 8V,\ f_m = 2000Hz,\ f_c = 2 \times 10^6 Hz,\ A_c = 4V$ we have

$$\begin{aligned} S_{FM}(t) =& A_c \cos[2\pi f_c t + 2\pi k_f \int_{-\infty}^{t} m(\eta) d(\eta)] \\ =& 4\cos[2\pi \times 2 \times 10^6 t - 2\pi \times 500 \frac{8}{2\pi 2000} \cos(2\pi \times 2000t)] \\ =& 4\cos[4 \times 10^6 \pi t - 2\cos(4000\pi t)]. \end{aligned} \tag{1}$$

Problem 6.3

Maximum Doppler Spread: $f_m = \frac{v}{\lambda} = \frac{80 \times 1.6 \times 10^3/3600}{300/440} = 52.1\text{Hz}$.

And the notch width is $2f_m = 104.2\text{Hz}$.

Problem 6.4

Interpreting W as f_m (max audio frequency) and f_d as Δ_f (frequency deviation) we get:

$\beta_f = \frac{f_d}{w} = \frac{12}{4} = 3$.

Problem 6.13

For SNR=30dB=1000, B=200KHz, the maximum possible data rate, $C = B\log_2\left(1 + \frac{S}{N}\right) = 1.99\text{Mbps}$.

The GSM data rate us 270.833kbps, which is only about 0.136C.

Problem 6

By applying the operator $\mathcal{F}(B) \triangleq \frac{\partial^2 B}{\partial x^2} + \frac{1}{x}\frac{\partial B}{\partial x} + B + \frac{1}{x^2}\frac{\partial^2 B}{\partial \theta^2}$ to the function $B(x,\theta) = e^{jx\sin(\theta)}$ we get

$$\begin{aligned}\mathcal{F}(B) &= -\sin^2(\theta)e^{jx\sin(\theta)} + j\frac{\sin(\theta)}{x}e^{jx\sin(\theta)} + e^{jx\sin(\theta)} + \frac{1}{x^2}(-jx\sin(\theta) - x^2\cos(\theta))e^{jx\sin(\theta)} \\ &= \left[-\sin^2(\theta) + j\frac{\sin(\theta)}{x} + 1 - j\frac{\sin(\theta)}{x} - \cos^2(\theta)\right]e^{jx\sin(\theta)} = 0\end{aligned} \tag{2}$$

By applying the same operator to $B(x,\theta) = \sum_n J_n(x)e^{jn\theta}$ we should get the same result:

$$\begin{aligned}\mathcal{F}(B) &= \sum_n \frac{\partial^2 J_n(x)}{\partial x^2}e^{jx\theta} + \frac{1}{x}\sum_n \frac{\partial J_n(x)}{\partial x}e^{jn\theta} + \sum_n J_n(x)e^{jn\theta} - \frac{1}{x^2}\sum_n n^2 J_n(x)e^{jn\theta} \\ &= \sum_n \left[\frac{\partial^2 J_n(x)}{\partial x^2} + \frac{1}{x}\frac{\partial J_n(x)}{\partial x} + J_n(x) - \frac{n^2}{x^2}J_n(x)\right]e^{jn\theta} = 0\end{aligned} \tag{3}$$

Therefore,

$$\frac{\partial^2 J_n(x)}{\partial x^2} + \frac{1}{x}\frac{\partial J_n(x)}{\partial x} + J_n(x) - \frac{n^2}{x^2}J_n(x) = 0 \tag{4}$$