COLUMBIA UNIVERSITY

Department of Electrical Engineering

EE E4703: Wireless Communications

Solution for Homework Assignment No. 4

Problem 1

Rappaport, p. 249, Problem 5.6

We must first find the first and second moment of the excess delay (and use only linear scale – not dB – to do it). We use them to get the delay spread.

For the indoor channel:

$$\bar{t} = \frac{\sum P(t_k) t_k}{\sum P(t_k)} = \frac{1\cdot 0 + 1\cdot 50 + 0.1\cdot 75 + 0.01\cdot 100}{1+1+0.1+0.01} = 27.73 ns$$

$$\overline{t^2} = \frac{\sum P(t_k) t_k^2}{\sum P(t_k)} = \frac{1\cdot 0^2 + 1\cdot 50^2 + 0.1\cdot 75^2 + 0.01\cdot 100^2}{1+1+0.1+0.01} = 1499\, ns^2$$

$$s_t = \sqrt{\overline{t^2} - \bar{t}^2} = 27.02 ns$$

It is given that:

$$s_t / T_s \le 0.1$$

$$\therefore T_s \ge 10 s_t = 270.2 ns$$

$$R_s = \frac{1}{T_s} \le 3.7 Msym/\sec$$

For the outdoor channel:

$$\bar{t} = \frac{\sum P(t_k) t_k}{\sum P(t_k)} = \frac{0.01\cdot 0 + 0.1\cdot 5 + 1\cdot 10}{0.01+0.1+1} = 9.46 ms$$

$$\overline{t^2} = \frac{\sum P(t_k) t_k^2}{\sum P(t_k)} = \frac{0.01\cdot 0^2 + 0.1\cdot 5^2 + 1\cdot 10^2}{0.01+0.1+1} = 92.34\, ms^2$$

$$s_t = \sqrt{\overline{t^2} - \bar{t}^2} = 1.69 ms$$

$$s_t / T_s \leq 0.1$$

$$\therefore T_s \geq 10 s_t = 16.9ms$$

$$R_s = \frac{1}{T_s} \leq 59.17 Ksym/\sec$$

Problem 2

Rappaport, p. 250, Problem 5.8

The Coherence bandwidths are given in equations 5.38 & 5.39 in the text:

$$B_{C,90\%} \approx \frac{1}{50 s_t}$$

$$B_{C,50\%} \approx \frac{1}{5 s_t}$$

Indoor channel:

$$s_t = 27.02 ns$$

$$B_{C,90\%} \approx \frac{1}{50 s_t} = 740 KHz$$

$$B_{C,50\%} \approx \frac{1}{5 s_t} = 7.4 MHz$$

Outdoor channel:

$$s_t = 1.69 ms$$

$$B_{C,90\%} \approx \frac{1}{50 s_t} = 11.83 KHz$$

$$B_{C,50\%} \approx \frac{1}{5 s_t} = 118.3 KHz$$

Problem 3

Rappaport, pp. 252-253, Problem 5.28

(a)

$$\bar{t} = \frac{\sum P(t_k) t_k}{\sum P(t_k)} = \frac{1 \cdot 0 + 0.1 \cdot 1 + 1 \cdot 2}{1 + 0.1 + 1} = 1ms$$

$$\overline{t^2} = \frac{\sum P(t_k) t_k^{\,2}}{\sum P(t_k)} = \frac{1 \cdot 0^2 + 0.1 \cdot 1^2 + 1 \cdot 2^2}{1 + 0.1 + 1} = 1.952\,ms^2$$

$$s_t = \sqrt{\overline{t^2} - \bar{t}^2} = 0.976\,ms$$

(b) The maximum delay spread (20dB) definition is given in page 199 in the text. It is defined as the time delay between the first arriving signal and the maximum delay at which a multipath component is within 20 dB of the strongest arriving multipath signal.

In our case, the last multipath component arrives 2μs after the first arriving signal, so the maximum delay spread is **2μs**.

(c)

$$T_s \geq 10 s_t = 9.76ms$$

$$Rs = \frac{1}{T_s} = 102.5 Ksym/\sec$$

(d) We are looking for the Doppler Coherence Time.

$$v = 30\, {}^{km}\!/_{hr} = 8.33\, {}^{m}\!/_{s}$$

$$l = \frac{c}{f} = 0.333m$$

$$\therefore f_m = \frac{v}{l} = 25Hz$$

$$T_C = \frac{0.423}{f_m} = 16.92ms$$

Problem 4

Rappaport, p. 250, Problem 5.11

Rayleigh distribution *pdf:*

$$f_r(r) = \frac{r}{s^2} e^{-\frac{r^2}{2s^2}}$$

$$p(r < R) = \int_0^R f_r(r)dr = \int_0^R \frac{r}{s^2} e^{-\frac{r^2}{2s^2}} dr = \frac{1}{2s^2} \int_0^R e^{-\frac{r^2}{2s^2}} 2rdr$$

We replace the integration variable with $u = r^2$

$$u = r^2$$

$$du = 2rdr$$

$$p(r < R) = \frac{1}{2s^2} \int_0^{R^2} e^{-\frac{u}{2s^2}} du = \frac{1}{2s^2} 2s^2 e^{-\frac{u}{2s^2}} \Big|_{R^2}^{0} = 1 - e^{-\frac{R^2}{2s^2}}$$

The rms value (square root of the second moment) of a Rayleigh distribution is

$\sqrt{2s^2} = \sqrt{2}s$ and 10 dB **below** the rms value, **in amplitude**, is $\sqrt{\frac{2}{10}}s$.

$$p(r < \sqrt{0.2}s) = 1 - e^{-\frac{\left(\sqrt{0.2}s\right)^2}{2s^2}} = 1 - e^{-0.1} = 0.0952 = 9.52\%$$

Problem 5

$A = 15$

$B = 10$

$a = 3000$

$b = 10^4$

$w = 2p \cdot 10^6$

$$v(t) = (A + B\sin 2pat + C\cos 2pbt)\cos wt$$

$$= A\cos wt + B\sin 2pat\cos wt + C\cos 2pbt\cos wt$$

$$= A\cos wt + \frac{B}{2}\left[\sin(w + 2pa)t - \sin(w - 2pa)t\right] + \frac{C}{2}\left[\cos(w + 2pb)t + \cos(w - 2pb)t\right]$$

(a+b) The modulated wave will have frequency components at the following frequencies:

Amplitude	**Frequency [Hz]**
$\lvert A \rvert = 15$	$\frac{w}{2p} = 1MHz$
$\left\lvert \frac{B}{2} \right\rvert = 5$	$\frac{w}{2p} + a = 1.003\,MHz$
$\left\lvert \frac{B}{2} \right\rvert = 5$	$\frac{w}{2p} - a = 0.997\,MHz$
$\left\lvert \frac{C}{2} \right\rvert = 4$	$\frac{w}{2p} + b = 1.01MHz$
$\left\lvert \frac{C}{2} \right\rvert = 4$	$\frac{w}{2p} - b = 0.99\,MHz$