COLUMBIA UNIVERSITY Department of Electrical Engineering EE

EE4703: Wireless Communications

Solution for Homework Assignment No. 3

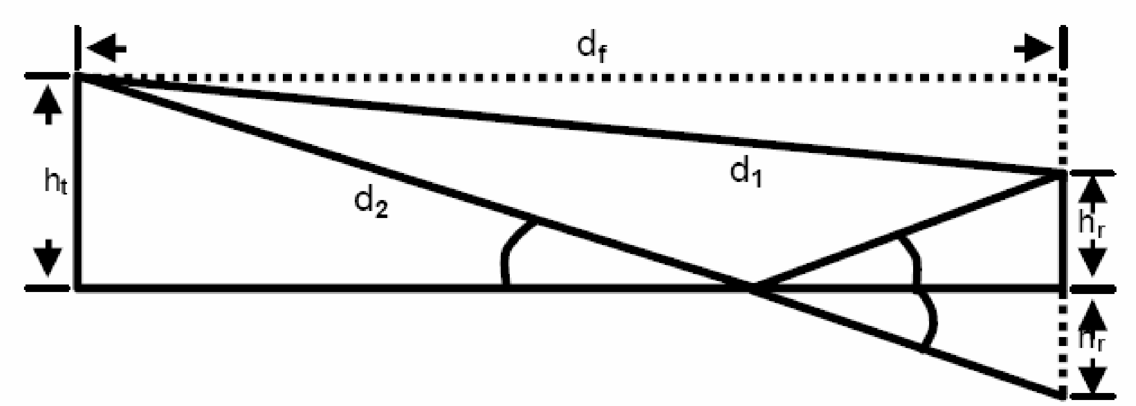
Problem 1 Rappaport, p. 169, Problem 4.15

Let d_f be the physical T-R separation.

d_1 is the free space propagation path length.

d_2 is the ground-reflected path length.

The first Fresnel zone is defined by a d_f that satisfies $d_2 = d_1 + \lambda/2$.



Note that d_2 is drawn using the reflection of h_t to make the calculation easier. Also note that in this drawing $h_t > h_r$, but the same calculation applies for $h_t < h_r$.

$$d_1 = \sqrt{d_f^{\,2} + (h_t - h_r)^2}$$
$$d_2 = \sqrt{d_f^{\,2} + (h_t + h_r)^2}$$

$$\sqrt{d_f^{\,2} + (h_t + h_r)^2} = \sqrt{d_f^{\,2} + (h_t - h_r)^2} + \frac{\lambda}{2}$$
$$d_f^{\,2} + (h_t + h_r)^2 = d_f^{\,2} + (h_t - h_r)^2 + \frac{\lambda^2}{4} + \lambda\sqrt{d_f^{\,2} + (h_t - h_r)^2}$$

$$2h_t h_r = -2h_t h_r + \frac{l^2}{4} + l\sqrt{d_f^{\,2} + (h_t - h_r)^2}$$

$$d_f^{\,2} + (h_t - h_r)^2 = \frac{1}{l^2}\left(4h_t h_r - \frac{l^2}{4}\right)^2$$

$$d_f^{\,2} = \frac{1}{l^2}\left(16h_t^{\,2} h_r^{\,2} - 2h_t h_r l^2 + \frac{l^4}{16}\right) - \left(h_t^{\,2} - 2h_t h_r + h_r^{\,2}\right)$$

$$d_f^{\,2} = \frac{1}{l^2}\left(16h_t^{\,2} h_r^{\,2} + \frac{l^4}{16} - h_t^{\,2} l^2 - h_r^{\,2} l^2\right) = \frac{1}{l^2}\left(\frac{l^2}{4} - 4h_t^{\,2}\right)\left(\frac{l^2}{4} - 4h_r^{\,2}\right)$$

$$d_f = \frac{\sqrt{\left(l^2 - 16h_t^{\,2}\right)\left(l^2 - 16h_r^{\,2}\right)}}{4l}$$

Had we used the parallel ray approximation, we would have received:

$$S = d_2 - d_1 = \frac{2h_t h_r}{d_f} = \frac{l}{2}$$

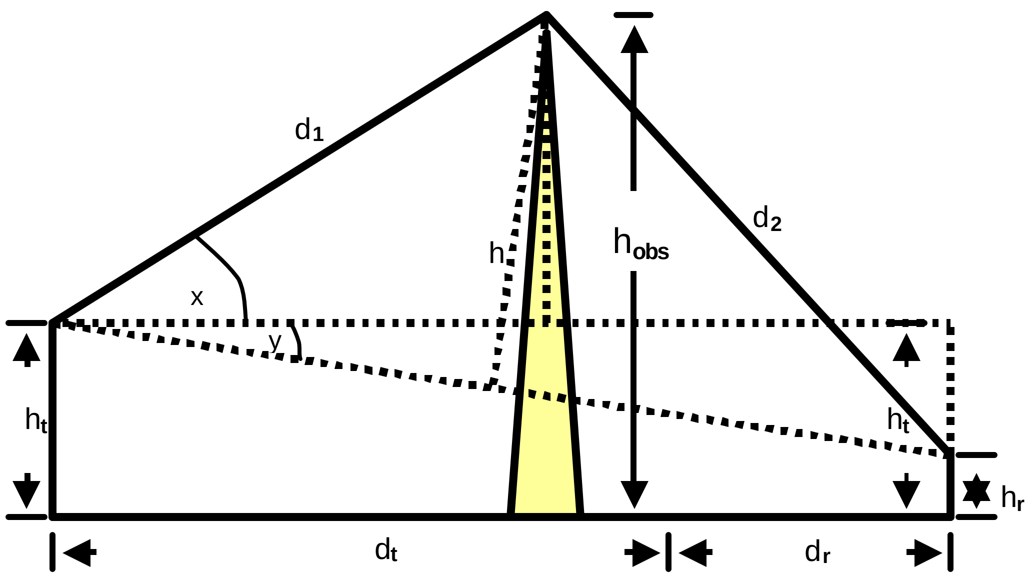
$$d_f = \frac{4h_t h_r}{l}$$

Whichis correct only if the carrier wavelength is very small in comparison with the receiver and transmitter heights.

Problem 2

Rappaport, p. 170, Problem 4.19

$$P_t = 10w = 40dBm$$
$$G_t = 10dB$$
$$G_r = 3dB$$
$$L = 1dB @ 900MHz$$
$$f = 900Mhz$$
$$\therefore l = c/f = \frac{1}{3}m$$



$$h_t = 60m$$
$$d_t = 3000m$$
$$h_r = 5m$$
$$d_r = 2000m$$
$$h_{obs} = 400m$$

First, let’s compute the free-space model:

$$PL=\frac{l^2}{[4p(d_t+d_r)]^2}=2.81\cdot 10^{-11}=-105.5dB$$

$$P_{rec,dBm}=P_{tr,dBm}+PL+G_{t,dB}+G_{r,dB}-L_{dB}=40-105.5+10+3-1=-53.5dBm=4.47\cdot 10^{-9}w$$

We should extract h from the geometry of the problem

$$d_1=\sqrt{{d_t}^2+(h_{obs}-h_t)^2}=3019m$$

$$x=\arctan(\frac{h_{obs}-h_t}{d_t})=6.47^\circ$$

$$y=\arctan(\frac{h_t-h_r}{d_t+d_r})=0.63^\circ$$

$$h=d_1\cdot\sin(x+y)=373.15m$$

$$v=h\cdot\sqrt{\frac{2}{l}\left(\frac{1}{d_1}+\frac{1}{d_2}\right)}=26.39$$

$$G_{d,dB}=20\log\left(\frac{0.225}{v}\right)=-41.39dB$$

And then, the received power is:

$$P_{rec}=P_{tr,dBm}+PL+G_{t,dB}+G_{r,dB}-L_{dB}+G_{d,dB}=-94.89dBm=3.24\cdot 10^{-13}w$$

The received power is ***$3.24\cdot 10^{-13}$ watts.***

Without the obstacle, it would have been ***$4.47\cdot 10^{-9}$ watts*****.**

The path loss caused by diffraction is ***41.39dB.***

Problem 3

Rappaport, p. 170, Problem 4.20

(a)

$$f = 50\,Mhz$$

$$\therefore\ l = {}^{c}\!/\!_{f} = 6m$$

$$PL = \frac{l^2}{[4p(d_t + d_r)]^2} = 9.12 \cdot 10^{-9} = -80.4dB$$

Without diffraction:

$$P_{rec,dBm} = P_{tr,dBm} + PL + G_{t,dB} + G_{r,dB} - L_{dB} = -28.4dBm = 1.45 \cdot 10^{-6}\,w$$

And with diffraction:

$$v = h \cdot \sqrt{\frac{2}{l}\left(\frac{1}{d_1} + \frac{1}{d_2}\right)} = 6.22$$

$$G_{d,dB} = 20\log\left(\frac{0.225}{v}\right) = -28.83dB$$

$$P_{rec,dBm} = P_{tr,dBm} + PL + G_{t,dB} + G_{r,dB} - L_{dB} + G_{d,dB} = -57.23dBm = 1.89 \cdot 10^{-9}\,w$$

The received power is ***1.89·10^{-9} watts*****.**

Without the obstacle, it would have been ***1.45·10^{-6} watts*****.**

The path loss caused by diffraction is ***28.83dB.***

(b)

$$f = 1900\,Mhz$$

$$\therefore\ l = {}^{c}\!/\!_{f} = 0.158m$$

$$PL = \frac{l^2}{[4p(d_t + d_r)]^2} = 6.24 \cdot 10^{-12} = -112.05dB$$

Without diffraction:

$$P_{rec,dBm} = P_{tr,dBm} + PL + G_{t,dB} + G_{r,dB} - L_{dB} = -60.05dBm = 9.89^{-10}\,w$$

And with diffraction:

$$v = h \cdot \sqrt{\frac{2}{l}\left(\frac{1}{d_1}+\frac{1}{d_2}\right)} = 38.32$$

$$G_{d,dB} = 20\log\left(\frac{0.225}{v}\right) = -44.62\,dB$$

$$P_{rec,dBm} = P_{tr,dBm} + PL + G_{t,dB} + G_{r,dB} - L_{dB} + G_{d,dB} = -104.67\,dBm = 3.41 \cdot 10^{-14}\,w$$

The received power is ***$3.41 \cdot 10^{-14}$ watts.***

Without the obstacle, it would have been ***$9.89 \cdot 10^{-10}$ watts.***

The path loss caused by diffraction is ***44.62dB.***

Problem 4

Rappaport, p. 170, Problem 4.22

$P_0 = 1mw = 0dBm$

$d_0 = 1m$

$d = 10m$

$\bar{n} = 3.5$

We require 10% of the measurements to be stronger than *-25dBm*.

i.e. $\Pr\{P_{rec} > -25dBm\} = 0.9$

We should first find the mean path loss and then look it up in the Q-function table.

$$\overline{P_{rec}} = P_0 - 10 \cdot \bar{n} \cdot \log\left(\frac{d}{d_0}\right) = -35dB$$

$$Q\left(\frac{X-m}{s}\right) = Q\left(\frac{(-35)-(-25)}{s}\right) = Q\left(\frac{-10}{s}\right) = 0.9$$

$$Q\left(\frac{10}{s}\right) = 0.1 \rightarrow \frac{10}{s} = 1.28$$

$s = 7.81dB$

The standard deviation would be 7.81 dB

Problem 5

Rappaport, p. 172, Problem 4.27

$P_{tr} = 15w = 41.76dBm$

$G_t = 12dB = 15.84$

$G_r = 3db = 2$

$BW_r = 30kHz$

$F_{rec} = 8dB$

$f_c = 1800 MHz$

$\therefore\ l = 0.167m$

$n = 4$

$s = 8dB$

$d_0 = 1000m$

We require 20 dB SNR 95% percent of the time

i.e. $\Pr\{SNR > 20dB\} = 0.95$

What is the maximum T-R separation that will ensure that SNR?

We first compute the noise threshold, and then the required received power.

Thermal noise spectral density at room temperature:

$$KT = -204dB\frac{w}{Hz} = 3.98 \cdot 10^{-21}\frac{w}{Hz}$$

So the thermal noise at the receives is

$$KT \cdot BW_r = 1.194 \cdot 10^{-16} = -129.2dBm$$

And the noise floor is:

$$F_{rec} + KT \cdot BW_r = -121.2dBm$$

Regarding the SNR:

$$\Pr\{SNR > 20dB\} = Q\left(\frac{20 - \overline{SNR}}{8}\right) = 0.95$$

$$Q\left(\frac{\overline{SNR} - 20}{8}\right) = 1 - 0.95 = 0.05$$

$$\frac{\overline{SNR} - 20}{8} = 1.645$$

$$\overline{SNR} = 33.16dB$$

The required received power will be:

$$P_{rec,\min} = -121.2 + 33.16 = -88.04dBm$$

The Power at d_0:

$$P_0 = \frac{P_{tr} G_t G_r l^2}{(4pd_0)^2} = 8.39 \cdot 10^{-8} w = -40.76\, dBm$$

$$P_{rec,\min} = P_0 - 10n \cdot \log\left(\frac{d_{\max}}{d_0}\right)$$

$$\therefore d_{\max} = d_0 \cdot 10^{\frac{P_{0,dBm} - P_{rec,\min}}{10n}} = 15.2km$$

Problem 6

Rappaport, p. 172, Problem 4.28

$SNR_{req} = 25dB$

$f_c = 900\,MHz$

$\therefore\ l = 0.333m$

$EIRP = P_{tr}G_t = 100W = 50\,dBm$

$G_r = 0dB = 1$

$AMPS \rightarrow BW = 30\,KHz$

$F = 10dB$

$d = 10km$

$\bar{n} = 4$

$s = 8dB$

$d_0 = 1km$

We will find P_0, and then the average received power.

$$P_0 = \frac{P_{tr}G_tG_r\,l^2}{(4pd_0)^2} = \frac{EIRP \cdot G_r\,l^2}{(4pd_0)^2} = 7.02 \cdot 10^{-8}\,w = -41.54dBm$$

$$\overline{P_{rec}} = P_0 - 10\bar{n} \cdot \log\left(\frac{d}{d_0}\right) = -81.54dBm$$

The thermal noise, just like in problem 4.27, is:

$$KT \cdot BW_r = 1.194 \cdot 10^{-16} = -129.2dBm$$

And the noise floor

$$NF = F_{rec} + KT \cdot BW_r = -119.2dBm$$

The mean SNR:

$$\overline{SNR} = \overline{P_{rec}} - NF = 37.66\,dB$$

$$\Pr\{SNR > 25dB\} = Q\left(\frac{25dB - \overline{SNR}}{s}\right) = Q(-1.58) = 1 - Q(1.58) = 0.943$$

The required SNR will be achieved 94.3% of the time.