COLUMBIA UNIVERSITY Department of Electrical Engineering EE E4703:

Wireless Communications

Solution for Homework Assignment No. 2

Problem 1
Rappaport, p. 168, Problem 4.1

$P_t = 10w$

$G_t = 0dB = 1$

$G_r = 0dB = 1$

$f_c = 900MHz$

$r = 1km = 1000m$

$$\lambda = \frac{c}{f_c} = \frac{3 \cdot 10^8 \, m/s}{900MHz} = 0.333m$$

$$P_{rec} = P_{tr} \frac{G_t G_r \lambda^2}{(4\pi r)^2} = 7.02 \cdot 10^{-9} w$$

The received power is $7.02 \cdot 10^{-9}$ watts.

Problem 2

Find the directivity $D = D(n)$ of an antenna pattern described by $dP/d\Omega = \cos^n\theta$, forward only (i.e., for $0 \le \theta \le \pi/2$ but for all azimuths $0 \le \varphi < 2\pi$). The answer should be given as a function of the integer n .

$$\frac{dP}{d\Omega} = \cos^n \theta$$

We'll find the directivity function $D(\theta, \varphi)$ and then maximize it to find the directivity D.

$$D(\theta, \varphi) = \frac{4\pi}{P} \frac{dP}{d\Omega}$$

$$P = \iint \left(\frac{dP}{d\Omega} \right) d\Omega = \int_{\theta=0}^{\pi/2} \int_{\varphi=0}^{2\pi} \cos^n \theta \sin \theta \, d\theta \, d\varphi = 2\pi \int_{\theta=0}^{\pi/2} \cos^n \theta \sin \theta \, d\theta$$

$$u = \cos\theta$$

$$du = -\sin\theta \, d\theta$$

$$P = 2\pi \int_{u=0}^{1} u^n du = 2\pi \left. \frac{u^{n+1}}{n+1} \right|_{u=0}^{1} = \frac{2\pi}{n+1}$$

$$D(\theta, \varphi) = \frac{4\pi}{P} \frac{dP}{d\Omega} = 2(n+1)\cos^n \theta$$

$$D(n) = \max D(\theta, \varphi) = 2(n+1)$$

The directivity of the antenna is D(n)=2(n+1).

Problem 3

Suppose we succeeded in generating a sectoral radiation pattern for a wireless cell that were described as follows. It is uniform (i.e., a constant) in all directions within both the horizontal sector 0° < j < 120° and in the range 80° < q < 90° of elevation from the vertical; it is zero for all directions outside these intervals. What is the directivity (in dB) of this pattern?

Let's assume we have R w/rad^2 radiation in the given sector.
We shall integrate it over the sector to find the emitted power.

First, convert the angles to radians:

$$0 < j < 2p/3$$

$$4p/9 < q < p/2$$

$$\frac{dP}{d\Omega}_{,\max} = R$$

$$D(q,j) = \frac{4p}{P}\frac{dP}{d\Omega}$$

$$P = \iint \left(\frac{dP}{d\Omega}\right) d\Omega = \int_{q=\frac{4p}{9}}^{p/2} \int_{j=0}^{\frac{2p}{3}} R \sin q \, dq \, dj = R \cdot \frac{2p}{3} \cdot \cos q \Big|_{\frac{p}{2}}^{\frac{4p}{9}} = 0.3637R$$

$$D = D(q,j)_{,\max} = \frac{4p}{P}\frac{dP}{d\Omega}_{,\max} = \frac{4p}{0.3637R}R = 34.55 = 15.38dB$$

The directivity of this radiation pattern is 15.38 dB.

Problem 4

The CTS satellite has an 11.7 GHz transmitter on board, which provides 200 mW to its 19.3 dB gain antenna. What power (in watts) can be received by a ground station antenna (a 3.66 m diameter parabolic reflector) with 50.4 dB gain? The satellite is in synchronous orbit 36,941 km above the ground station.

$$f = 11.7Ghz$$

$$\therefore l = {}^{c}\!/_{f} = 0.0256m$$

$$P_t = 0.2w$$

$$G_t = 19.3dB = 85.11$$

$$G_r = 50.4dB = 109648$$

$$r = 36{,}941km = 36.941 \cdot 10^6 m$$

For a satellite-to-ground problem we can apply the free space communications model

$$P_{rec} = P_{tr} \frac{G_t G_r l^2}{(4pr)^2} = 5.676 \cdot 10^{-15} w$$

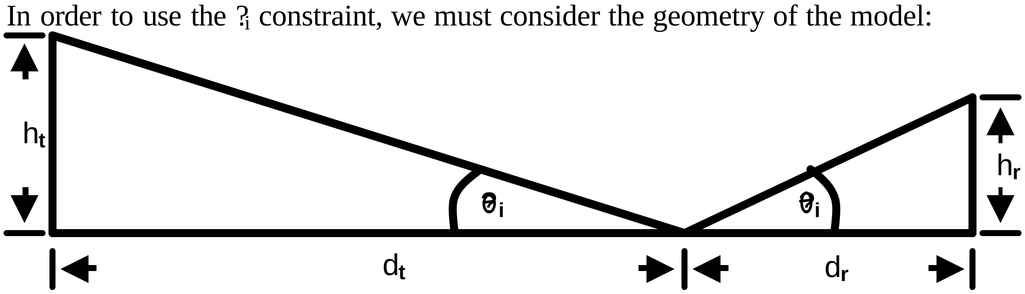
The received power is 5.676·10^{-15} watts.

Problem 5
Rappaport, p. 168, Problem 4.10

$$h_r = 2\,m$$
$$\theta_{\Delta,\min} = 6.261\ rad$$
$$\theta_{i,\max} = 5^\circ$$
$$f = 900\ Mhz$$
$$\therefore\ \lambda = c/f = 0.333\ m$$

$$\theta_\Delta = \frac{2\pi\Delta}{\lambda} = \frac{2\pi}{\lambda}\frac{2h_t h_r}{d} = \frac{4\pi h_t h_r}{\lambda d} = 75.473\ \frac{h_t}{d}$$
$$\theta_{\Delta,\min} = 75.473\ \frac{h_{t,\min}}{d_{\min}} = 6.261$$

In order to use the θ_i constraint, we must consider the geometry of the model:



$$\frac{h_r}{d_r} = \tan\theta_i = \tan 5^\circ$$
$$d_r = \frac{h_r}{\tan 5^\circ} = 22.86m$$
$$\frac{h_t}{d_t} = \frac{h_t}{d - d_r} = \frac{h_{t,\min}}{d_{\min} - 22.86} = \tan 5^\circ = 0.0875$$

$$75.473\frac{h_{t,\min}}{d_{\min}} = 6.261$$
$$\frac{h_{t,\min}}{d_{\min} - 22.86} = 0.0875$$

Solve the 2 equations for $d_{\min}$ and $h_{t,\min}$

$$d_{\min} = 443.5m$$
$$h_{t,\min} = 36.8m$$

Problem 6
Rappaport, p. 168, Problem 4.11

To completely cancel each other, the signals must have a 2p phase difference (or an integral multiple of 2p).

$$q_{\Delta} = \frac{2p}{l} s = \frac{2p}{l} \frac{2h_r h_t}{d} = 2pn$$

$n = 1,2,3....$

$$\therefore d_0 = \frac{2h_r h_t}{ln}$$

The signal nulls will occur when d=d_0, above.