COLUMBIA UNIVERSITY
Department of Electrical Engineering
EE E4703: Wireless Communications

Solution for Homework Assignment No. 1

Problem 1.7, p 22
A pager has the longest battery life (lowest power consumption) as it is only used as a receiver and does not transmit at all.

Problem 1.8, p 22
A cellular phone has the shortest battery life (highest power consumption) as it transmits to a much bigger range than a cordless phone.

Problem 1.9, p 22

One 3-minute call, every day. The total charge consumption for one day:

$$Q_{daily} = I_{idle} \cdot t_{idle} + I_{call} \cdot t_{call} = \left(24h - \frac{1}{20}h\right) \cdot 35mA + \frac{1}{20}h \cdot 250mA = 850.75mA \cdot h$$

Given a 1 $A \cdot h$ battery,

$$\frac{1A \cdot h}{850.75 \frac{mA \cdot h}{day}} = 1.175 days = 28.2h$$

The battery will last for **28 hours and 12 minutes**.

Every 6 hours (4 calls per day):

$$Q_{daily} = I_{idle} \cdot t_{idle} + I_{call} \cdot t_{call} = \left(24h - \frac{4}{20}h\right) \cdot 35mA + \frac{4}{20}h \cdot 250mA = 883mA \cdot h$$

$$\frac{1A \cdot h}{883 \frac{mA \cdot h}{day}} = 1.133 days = 27.18h$$

The battery will last for **27 hours and 11 minutes**.

Every hour (24 calls per day):

$$Q_{daily} = I_{idle} \cdot t_{idle} + I_{call} \cdot t_{call} = \left(24h - \frac{24}{20}h\right) \cdot 35mA + \frac{24}{20}h \cdot 250mA = 1098mA \cdot h$$

$$\frac{1A \cdot h}{1098 \frac{mA \cdot h}{day}} = 0.911 days = 21.86h$$

The battery will last for **21 hours and 52 minutes**.

Maximum talk time ?

$I_{call} = 250mA$

$$\frac{1A \cdot h}{250mA} = 4h$$

The maximum talk time would be **4 hours**:

Problem 3.4, p 97

(a) Two simplex channels make one duplex channel. Therefore, each duplex channel has a 2·25=50 kHz RF Bandwidth.

20 MHz can be divided into 20MHz/50kHz = **400 duplex channels.**

(b) Frequency reuse factor of 4 (N=4), means that the 400 hundred channels will be divided among the 4 cells of every cluster (and then reused in every cluster). Therefore **100 channels per cell site** will be used.

Problem 3.16, p 100

Given:

P_0= 1 mW = 0 dBm

d_0= 1 m

n= 3

Required:

P_r<-100 dBm

$$P_r = P_0 \left(\frac{D}{d_0} \right)^{-n}$$

$$P_r(dBm) = P_r(dBm) - 10n \log\left(\frac{d}{d_0} \right) < -100dBm$$

Rearrange:

$$D > 10^{\frac{-100dBm - 0dBm}{-10n}} \cdot d_0 = 10^{\frac{10}{3}} \cdot 1m = 2154m$$

For N=7:

$$R = \frac{D}{\sqrt{3N}} > 470m$$

For N=4:

$$R = \frac{D}{\sqrt{3N}} > 622m$$

Problem 3.26, p 101-102

$\lambda = 3 \, {}^{calls}\!/\!{}_{hr}$

$\frac{1}{\mu} = 5 \, {}^{\min}\!/\!{}_{call} = \frac{1}{12} \, {}^{hr}\!/\!{}_{call}$

(a) $A_u = \lambda / \mu = \frac{1}{4} erlang$

(b) C=1.

$$erlb(A,1) = \frac{A}{1+A} = 0.01$$
$$\therefore A = 0.0101$$

Using the Eralng B formula it seems as if the maximum traffic intensity that will achieve the required blocking probability is 0.0101, well below the traffic intensity of a single user. However, if there is only one user, he will never be blocked.

Therefore the answer is **one user**.

(c) C=5.

$$erlb(A,5) = 0.01$$
$$\therefore A = 1.361$$
$$U = \left\lfloor {}^{A}\!/\!{}_{A_u} \right\rfloor = 5 users$$

Note: *the symbol* $\lfloor x \rfloor$ *denotes "the greatest integer value less or equal to x".*

(d) Now, U = 10 users.

$$A = U \cdot A_u = 10 \cdot 0.25 = 2.5 erlang$$
$$GOS = \Pr\{block\} = erlb(2.5,5) = 0.0697 \approx 7\%$$

This GOS means that **7% of the calls will be blocked**. Such performance is unacceptable in cellular systems. Usually the GOS is kept below 2%.

Problem 3.15, p 100

(a) $GOS = \Pr\{block\} = 0.02$

Channels	Maximum System Capacity	Maximum Capacity per User
C=4	erlb (A,4)=0.02 A= 1.09	A_C=A/4=0.273
C=20	erlb (A,20)=0.02 A= 13.18	A_C=A/20=0.659
C=40	erlb (A,40)=0.02 A= 31	A_C=A/40=0.775

(b)

$$C = 40$$
$$\lambda = 1 \,{}^{calls}\!/_{hr}$$
$$\frac{1}{\mu} = 105 \,{}^{\sec}\!/_{call} = 0.0292 \,{}^{hr}\!/_{call}$$

From (a), A=31.

$$\lambda = 1 \,{}^{calls}\!/_{hr}$$
$$\frac{1}{\mu} = 105 \,{}^{\sec}\!/_{call} = 0.0292 \,{}^{hr}\!/_{call}$$
$$A_u = {}^{\lambda}\!/_{\mu} = 0.0292 erlang$$
$$U = \left\lfloor {}^{A}\!/_{A_u} \right\rfloor = 1061 users$$

1061 users can be supported.

Note: *the symbol* $\lfloor x \rfloor$ *denotes "the greatest integer value less or equal to x".*

(c) For an LCD (lost call delayed) system:

$$\Pr\{delay > t\} = erlc(A,C)\cdot e^{-\mu(C-A)t}$$

$$t = 20\,\text{sec}$$

$$\frac{1}{\mu} = 105\,{}^{\text{sec}}\!/_{call}$$

$$C = 4$$

$$A = 1.09\,erlang$$

$$\Pr\{delay > 20\} = erlc(1.09,4)\cdot e^{-0.554} = 0.0156 = 1.56\%$$

$$C = 20$$

$$A = 13.18\,erlang$$

$$\Pr\{delay > 20\} = erlc(13.18,20)\cdot e^{-1.29} = 0.0154 = 1.54\%$$

$$C = 40$$

$$A = 31\,erlang$$

$$\Pr\{delay > 20\} = erlc(31,40)\cdot e^{-1.71} = 0.01499 = 1.499\%$$

(d) Comparing (a) and (c), we can see that the LCD system, with a 20 second queue performs better than the LCC system. We can see that while the LCC system assures a 2% GOS, the LCD system gives a lower (and thus better) GOS of 1.5%.

Problem 3.17, p 101

From 3.16, we know that for N=7, R=470m.

The area of a (hexagonal) cell is therefore:

$$R_{cell} = 0.47km$$
$$U_{per-km^2} = 9000$$
$$\lambda = 1\, {}^{call}\!/\!_{hr}$$
$$\frac{1}{\mu} = 1\, {}^{\min}\!/\!_{call} = \tfrac{1}{60}\, {}^{hr}\!/\!_{call}$$
$$C = 90$$
$$AREA_{cell} = \frac{3\sqrt{3}}{2} R_{cell}{}^{2} = 0.574km^2$$
$$U_{cell} = U_{per-km^2} \cdot AREA_{cell} = 5166users$$
$$A_u = \lambda\!/\!_{\mu} = \tfrac{1}{60}\, erlang$$
$$A = UA_u = 86.1erlang$$

$$GOS = \Pr\{delay > 20\} = erlc(A,C)\, e^{-\mu(C-A)t} = erlc(86.1,90)\, e^{-\frac{(90-86.1)20}{60}} = 0.1575 = 15.75\%$$