COLUMBIA UNIVERSITY Department of Electrical Engineering EE E4703: Wireless Communications

Solution for Homework Assignment No. 6

Problem 1

The decision points don't always correspond to the peaks of the envelope because the signal is the sum of a few pulses. Although these pulses are designed to have zero-crossings in every integer except zero, and thus have zero ISI at the decision points, the values between the decision points represent the sum of the pulses which isn't necessarily zero. In this figure we can see that the side-lobe of p(t-6T) rises faster than the main lobe of p(t-5T) falls. This causes the sum of the two to be higher than the max value of p(t-5T).

Problem 2

The ISI will be the ratio of the interference amplitude over the signal amplitude. As the pulse is Gaussian we can assume that only the two adjacent pulses will contribute significantly to the ISI (the contribution of the other pulses can be neglected).

If the pulse is $p(t) = e^{\frac{\pi t^2}{T^2}}$, then at time t=kT, the values of the signal pulse and the two adjacent pulses will be:

$$p(kT - kT) = p(0) = e^{0} = 1$$

$$p\left[(k+1)T - kT\right] = p(T) = e^{-\frac{\pi T^2}{T^2}} = e^{-\pi}$$

$$p\left[(k-1)T - kT\right] = p(-T) = e^{-\frac{\pi(-T)^2}{T^2}} = e^{-\pi}$$

$$ISI = \frac{p(T) + p(-T)}{p(0)} = 2e^{-\pi} = 0.086 = -21.3dB$$

Problem 3

(a) ASK:

$$\Pr\{e\} = Q\left(\sqrt{\frac{E_b}{2N_0}}\right) = 10^{-6}$$

$$SNR = \frac{E_b}{N_0} = 2\left[Q^{-1}\left(10^{-6}\right)\right]^2 = 2 \cdot 4.75^2 = 45.125 = 16.54dB$$

(b) FSK:

$$\Pr\{e\} = Q\left(\sqrt{\frac{E_b}{N_0}}\right) = 10^{-6}$$

$$SNR = \frac{E_b}{N_0} = \left[Q^{-1}\left(10^{-6}\right)\right]^2 = 4.75^2 = 22.563 = 13.53dB$$

(c) BPSK:

$$\Pr\{e\} = Q\left(\sqrt{\frac{2E_b}{N_0}}\right) = 10^{-6}$$

$$SNR = \frac{E_b}{N_0} = \frac{\left[Q^{-1}\left(10^{-6}\right)\right]^2}{} = \frac{4.75^2}{2} = 11.281 = 10.52dB$$

(d) 8-PSK (m=8):

$$\Pr\{e\} = 2Q\left(\sqrt{\frac{E_b}{N_0} \cdot 2\log_2 M}\ \sin\left(\frac{\pi}{M}\right)\right) = 10^{-6}$$

$$SNR = \frac{E_b}{N_0} = \frac{\left[Q^{-1}\left(\frac{10^{-6}}{2}\right)\right]^2}{\left(\sin\left(\frac{\pi}{M}\right)\right)^2 2\log_2 M} = \frac{\left[Q^{-1}\left(5 \cdot 10^{-7}\right)\right]^2}{0.879} = \frac{4.892^2}{0.879} = 27.226 = 14.34dB$$

Problem 4

$$R_S = 10 \cdot 10^3 bps$$

$$T_C = 0.1 \mu s$$

$$\therefore R_C = \frac{1}{0.1 \cdot 10^{-6} s} = 10 \cdot 10^6 bps$$

$$PG = \frac{R_C}{R_S} = 1000$$
