Lecture-3
Junction Diode Characteristics
Part-I

1. **Potential Variation with in a Graded Semiconductor:** Consider a semiconductor shown in the figure-1(a). Here the concentration is the function of x. This type of doping is called "non-uniform" or "graded". As there is no external voltage applied, the total current must be equal to zero. But due to the concentration gradient of p along the x-axis, there exists a non-zero diffusion current. So in order the total current to be zero, there must exist an equal and opposite current in the form of the drift current. Thus from the equation (21) of lecture notes-2, we have

$$
\begin{aligned}
J_p &= q\mu_p p\varepsilon - qD_p\frac{dp}{dx} \\
\Rightarrow J_p &= q\mu_p p\varepsilon - qD_p\frac{dp}{dx} = 0 \\
\Rightarrow D_p\frac{dp}{dx} &= \mu_p\varepsilon p \\
\Rightarrow \frac{D_p}{\mu_p}\frac{dp}{dx} &= p\varepsilon \qquad (1)
\end{aligned}
$$

Using the definition of electric field $\varepsilon = -dV/dx$ and the Eqn.(17) of lecture notes -2, Eqn.(1) in the above becomes,

$$
\begin{aligned}
\frac{V_T}{p}\frac{dp}{dx} &= -\frac{dV}{dx} \\
\Rightarrow dV &= -V_T\frac{dp}{p} \\
\Rightarrow \int_{V_1}^{V_2} dV &= -V_T\int_{p_1}^{p_2}\frac{1}{p}dp \\
\Rightarrow V_2 - V_2 \equiv V_{21} &= -V_T \times \{ln(p)\}_{p1}^{p2} \\
\Rightarrow V_{21} &= V_T \times ln\left(\frac{p_1}{p_2}\right) \qquad (2)
\end{aligned}
$$

Thus from the Eqn.(2) we get the concentration of the holes at $x = x_1$, by taking anti-log on both sides,

$$p_1 = p_2 e^{\frac{V_{21}}{V_T}} \qquad (3)$$

Similarly the concentration of the electrons at $x = x_1$ and x_2 is given as

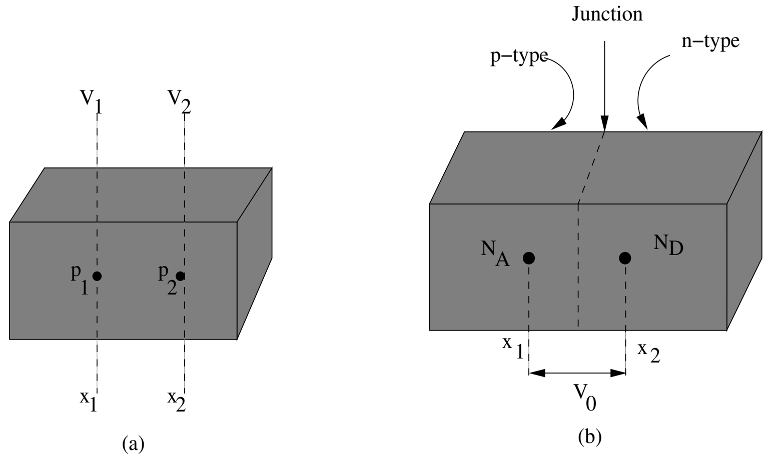

Figure 1: (a)Graded Semiconductor:$p(x)$ is not constant; (b) Step-Graded p-n junction

follows,

$$n_1 = n_2 e^{\frac{-V_{21}}{V_T}} \tag{4}$$

Taking the product of Eqn.(3) and (4) we get,

$$n_1 p_1 = n_2 p_2 \tag{5}$$

Mass action law can be verified from the Eqn.(5), by substituting $n = p = n_i$, which becomes,

$$np = n_i^2$$

2. **Open Circuited Step Graded Junction:** Consider the semiconductor as shown in the figure-1(b). The left half of the bar is p-type with the concentration N_A, whereas the right part is the n-type with uniform density N_D. The dashed plane is the metallurgical (p-n) junction separating the two different concentration. This type of doping, where the density changes abruptly from p-type to n-type is called “step grading”. The step graded junction is located at a plane where concentration is zero. There exists the potential called “contact difference of potential” V_0 given by,

$$V_0 = V_T \times ln\left(\frac{p_{p_0}}{p_{n_0}}\right) \tag{6}$$

where p_{p_0} is the thermal equilibrium concentration of holes in the p-type and

p_{n_0} is the thermal equilibrium concentration of holes in the n-type. Note that this equation is obtained from Eqn.(2) by substituting $p_1 = p_{p_0}$, $p_2 = p_{n_0}$ and $V_{21} = V_0$. So p_{n_0} is the concentration of minority carriers, which can be calculated from the mass-action law, as given below. Thus we have,

$$p_{p_0} \approx N_D$$

$$p_{n_0} N_A = n_i^2 \Rightarrow p_{n_0} = \frac{n_i^2}{N_A}$$

Substituting these equations in the Eqn.(6) , we get

$$V_0 = V_T \times ln \frac{N_A N_D}{n_i^2} \tag{7}$$

Space-Charge Region: Because there is a density gradient across the junction, holes will initially diffuse to the right across the junction, and electrons to the left. We see that the positive holes which neutralized the acceptor ions near the junction in the p-type silicon have disappeared as a result of combination with electrons which have diffused across the junction. Similarly, the neutralizing electrons in the n-type silicon have combined with holes which have crossed the junction from the p material. The unneutralized ions in the neighborhood of the junction are referred to as uncovered charges. Since the region of the junction is depleted mobile charge, it is called the "*depletion region*", the "*space-charge region*", or the "*transition region*. The thickness of the region is of the order of the wavelength of visible light (0.5 micron). Within this very narrow space-charge layer there are no mobile carriers. To the left of this region the carrier concentration is $p \approx N_A$, and to its right it is $n \approx N_D$.

At the junction (i.e., at $x = 0$), the uncompensated charge must be equal, thus

$$\begin{aligned} qN_D A x_{n_0} &= qN_A A x_{p_0} \\ \Rightarrow N_D x_{n_0} &= N_A x_{p_0} \end{aligned} \tag{8}$$

where x_{p_0} is space charge region length on the p-side and x_{n_0} is space charge region the n-side, as shown in the figure-2. Thus the total length of the depletion region

$$W = x_{p_0} + x_{n_0} \tag{9}$$

Let us assume that the donor concentration is more that the acceptor, i.e., $N_D > N_A$. The depletion region penetrates more in to the lightly doped region rather than highly doped region, which will be justified at the end of the section. Therefore we have $x_{p_0} > x_{n_0}$. Now from the Poisson equation and

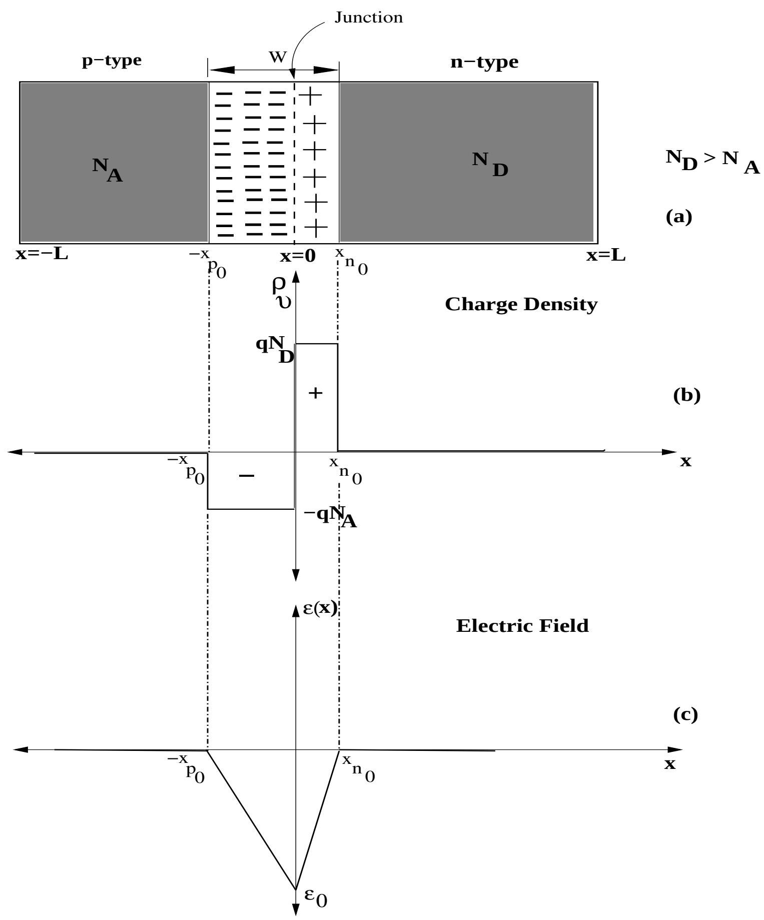

Figure 2: (a)Open Circuited p-n junction (b) Charge Density (ρ_v) (c) Electric Field (ε_0)

the definition of the electric field, we have

$$\begin{aligned}
\frac{d^2V}{dx^2} &= -\frac{\rho_v}{\epsilon} \\
\frac{d}{dx}\left(\frac{dV}{dx}\right) &= -\frac{\rho_v}{\epsilon} \\
\varepsilon = -\frac{dV}{dx} & \\
\Rightarrow \frac{d\varepsilon}{dx} = \frac{\rho_v}{\epsilon} & \qquad (10)
\end{aligned}$$

where ρ_v is the volume charge density, which is charge present per unit volume (nq=concentration of carriers per unit volume $\times$ charge of the carrier). Hence from the figure-2(b) we get

$$\rho_v = \begin{cases} -qN_A & \text{for } -x_{p_0} < x < 0 \\ qN_D & \text{for } 0 < x < x_{n_0} \\ 0 & \text{otherwise} \end{cases} \qquad (11)$$

Substituting Eqn.(11) in Eqn.(10), we get the derivative of the electric field as

$$\frac{d\varepsilon}{dx} = \begin{cases} \frac{-qN_A}{\epsilon} & \text{for } -x_{p_0} < x < 0 \\ \frac{qN_D}{\epsilon} & \text{for } 0 < x < x_{n_0} \\ 0 & \text{otherwise} \end{cases} \qquad (12)$$

Hence the expression for the electric field in the step-graded p-n junction can be obtained by integrating above Eqn.(12), both sides with respect to x, over the limits $-\infty$ to x. Consider for the range $-x_{p_0} < x < 0$,

$$\begin{aligned}
\varepsilon(x) &= \int_{-\infty}^{x} \frac{qN_A}{\epsilon} dx \\
&= \int_{-\infty}^{-x_{p0}} 0 \times dx + \int_{-x_{p0}}^{x} \frac{-qN_A}{\epsilon} dx \\
&= -\frac{qN_A}{\epsilon}(x + x_{p_0}) \qquad (13)
\end{aligned}$$

at $x = 0$

$$\varepsilon(0) = -\frac{qN_A x_{p_0}}{\epsilon} \equiv \varepsilon_0 \qquad (14)$$

and at $x = -x_{p0}$, we have $\varepsilon(-x_{p0}) = 0$. For the range $0 < x < x_{n0}$, we get

$$\begin{aligned} \varepsilon(x) &= \int_{-\infty}^{-x_{p0}} 0 \times dx + \int_{-x_{p0}}^{0} \frac{-qN_A}{\epsilon} dx + \int_0^x \frac{qN_D}{\epsilon} dx \\ &= -\frac{qN_A}{\epsilon} x_{p0} + \frac{qN_D}{\epsilon} x \end{aligned} \tag{15}$$

Using the Eqn.(8), Eqn.(14) becomes

$$-\frac{qN_A x_{p0}}{\epsilon} \equiv \varepsilon_0 \equiv -\frac{qN_D x_{n0}}{\epsilon} \tag{16}$$

Therefore $\varepsilon(x)$ by substituting Eqn.(16) for the range $0 < x < x_{n0}$ becomes,

$$\varepsilon(x) = \frac{qN_D}{\epsilon} x + \varepsilon_0 \tag{17}$$

at $x = 0$, $\varepsilon(0) = \varepsilon_0$ and at $x = x_{n0}$, we have $\varepsilon(x_{n0}) = \frac{qN_D x_{n0}}{\epsilon} + \varepsilon = 0$. Hence in summary, the equation for the electric field is given by,

$$\varepsilon = \begin{cases} -\frac{qN_A}{\epsilon}(x + x_{p_o}) & \text{for } -x_{p0} < x < 0 \\ \frac{qN_D}{\epsilon} x + \varepsilon_0 & \text{for } 0 < x < x_{n0} \\ 0 & \text{otherwise} \end{cases} \tag{18}$$

This electric field is plotted as shown in the figure-2(c). Having obtained the electric field the potential across the junction can be obtained by integrating the electric field at the junction over the interval $-x_{p0}$ to x_{n0}. Thus we have,

$$V_0 = -\int_{-x_{p0}}^{x_{n0}} \varepsilon(x) dx \tag{19}$$

The above integral can be easily calculated, by interpreting the integral as the area under the curve. Hence area in the figure-2(c) can be thought as the sum of areas of two right angled triangles[1]

$$\begin{aligned} V_0 &= -\left(\frac{1}{2} \times x_{p0} \times \varepsilon_0 + \frac{1}{2} \times x_{n0} \times \varepsilon_0 \right) \\ &= -\frac{1}{2} \varepsilon_0 (x_{p0} + x_{n0}) \end{aligned}$$

[1] Area of right angled triangle is $\frac{1}{2} \times b \times h$, where b is the base and h is the height of the triangle

$$= -\frac{1}{2}\varepsilon_0 W \text{ by sub. Eqn.(9)} \tag{20}$$

Using the Eqn.(8) and the definition for ε_0, the "contact difference potential" (V_0) can also be written as

$$V_0 = \frac{1}{2}\frac{qN_D x_{n_0}}{\epsilon}W \tag{21}$$

The expressions for the space-charge region on both sides of the p-n junction i.e., x_{p_0} x_{n_0} can be calculated in terms of W, N_D and N_A as follows,
from Eqn.(8) $W = x_{p_0} + x_{n_0}$ and from Eqn.(8), we have

$$x_{p_0} = \frac{x_{n_0} N_D}{N_A}$$

Thus, substituting the above equation in W, we get

$$x_{n_0}\left(1 + \frac{N_D}{N_A}\right) = W$$

Rearranging this equation we have,

$$x_{n_0} = W\left(\frac{N_A}{N_A + N_D}\right) = \frac{W}{1 + \frac{N_D}{N_A}} \tag{22}$$

Similarly we get

$$x_{p_0} = W\left(\frac{N_D}{N_A + N_D}\right) = \frac{W}{1 + \frac{N_A}{N_D}} \tag{23}$$

Thus if we have $N_D > N_A$ i.e., $N_D/N_A > 1$ then clearly from the above expressions it is obvious that $x_{n_0} < x_{p_0}$. Thus the statement that "*depletion region penetrates more in the lightly doped region than the heavily doped region*" is justified. Substituting the value of x_{n_0} from Eqn.(22) in Eqn.(21), we get

$$V_0 = \frac{1}{2}\frac{q}{\epsilon}\left(\frac{N_D N_A}{N_A + N_D}\right)W^2 \tag{24}$$

$$\Rightarrow W = \left(\frac{2\epsilon V_0}{q}\left(\frac{N_A + N_D}{N_A N_D}\right)\right)^{\frac{1}{2}}$$

$$\Rightarrow W = \sqrt{\frac{2\epsilon V_0}{q}}\left(\frac{1}{N_A} + \frac{1}{N_D}\right)^{\frac{1}{2}} \tag{25}$$

Thus in the step-graded semiconductor the depletion width is directly proportional to $\sqrt{V_0}$. In some cases, only the concentrations of the donor and acceptor atoms are known (i.e., V_0 is not given). Then the width of the depletion region can be calculated by substituting Eqn.(7) in above equation. Thus we have

$$W = \sqrt{\frac{2\epsilon}{q}\left(ln\left(\frac{N_A N_D}{n_i^2}\right)\right)}\left(\frac{1}{N_A}+\frac{1}{N_D}\right)^{\frac{1}{2}} \tag{26}$$

Exercise: Derive the following expressions,

$$x_{p_0} = \sqrt{\frac{2\epsilon V_0}{q}}\left(\frac{N_D}{N_A(N_D+N_A)}\right)^{\frac{1}{2}}$$

$$x_{n_0} = \sqrt{\frac{2\epsilon V_0}{q}}\left(\frac{N_A}{N_D(N_D+N_A)}\right)^{\frac{1}{2}}$$

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