Lecture-1
Definitions, Energy band Diagrams and Ohms Law

Constant	Symbol	Value
Electronic charge	q	1.602×10^{-19} C
Electronic mass	m	9.109×10^{-31} kg
Ratio of charge to mass of an electron	q/m	1.759×10^{11} C/kg
Mass of atom of unit atomic weight (hypothetical)	$\ldots$	1.660×10^{-27} kg
Mass of proton	m_p	1.673×10^{-27} kg
Ratio of proton to electron mass	m_p/m	1.837×10^{3}
Planck's constant	h	6.626×10^{-34} J-s
Boltzmann constant	$\bar{k}$	1.381×10^{-23} J/K^o
Boltzmann constant	k	8.620×10^{-5} eV/K^o
Stefan-Boltzmann constant	σ	5.670×10^{-8} W/$(m^2)(K^4)$
Avogadro's number	N_A	6.023×10^{23} molecules/mole
Gas constant	R	8.314 J/(deg)(mole)
Velocity of light	c	2.998×10^{8} m/s
Faraday's constant	F	9.649×10^{3} C/mole
Volume per mole	V_0	2.241×10^{-2} m^3
Acceleration of gravity	g	9.807 m/s^2
Permeability of free space	μ_0	1.257×10^{-6} H/m
Permittivity of free space	ϵ_0	8.849×10^{-12} F/m

Table 1: Important physical constants and their values

1. **Definitions:**

   (a) Charge of Electron: The charge or quantity of negative electricity and mass of the electrons have been found to be 1.6×10^{-19}C and 9.11×10^{-31}kg respectively.

   $$1q = 1.6 \times 10^{-19} C$$

   (b) Current: It is defined as the rate of change of charge. $\triangle I = \frac{\triangle Q}{\triangle T}$, i.e change in charge per unit time. If charge is function of time $q(t)$, then instantaneous current is defined as,

   $$i(t) = \frac{dq}{dt}$$

or $I = Q/T$, where T is the time taken for the charge to drift. Measured in Amperes
Q: Current of $1pA = 1 \times 10^{-12}$A represents motion of approximately how many number of electrons per second?
A: From the definition of the current we have $I = Q/T$,

$$\Rightarrow I = \frac{Nq}{T}$$

where N is the number of electrons.

$$\Rightarrow I = \frac{N}{T} \times q$$

where N/T represents the number of electrons per unit time (s^{-1}). Hence

$$1 \times 10^{-12} = \frac{N}{T} \times 1.6 \times 10^{-19}$$

Thus we have rate of electrons as $N/T = 6 \times 10^6\ s^{-1}$

(c) Coulomb's Law: The force of attraction or repulsion (F_a or F_r) between any two charges Q_1 and Q_2, which are separated by distance r, is directly proportional to the product of individual charges and inversely to the square of the distance (r) between the charges. Thus is have

$$F = k \times \frac{Q_1 Q_2}{r^2}$$

where k is the proportionality constant defined as

$$k = \frac{1}{4\pi\epsilon}$$

where ϵ = permittivity of the medium = $\epsilon_0\epsilon_r$, ϵ_0 is permittivity of free space, or vacuum given by

$$\epsilon_0 = \frac{1}{36\pi} \times 10^{-9}\ \text{F/m}$$

which equals 8.849×10^{-12} F/m. ϵ_r is relative permittivity. Thus we have

$$F_a \text{ or } F_r = \frac{1}{4\pi\epsilon}\frac{Q_1 Q_2}{r^2} \tag{1}$$

(d) Field Intensity E(or Electric Field ε): The force F (newtons) on a unit positive charge in a electric field is the electric field intensity ε at that point

$$F_r = \frac{Q \times 1}{4\pi\epsilon r^2}$$

$$\Rightarrow \left(\frac{F_r}{1}\right) = \frac{Q}{4\pi\epsilon r^2}$$

$$\Rightarrow \varepsilon = \frac{F_r}{1} = \frac{Q}{4\pi\epsilon r^2}$$

$$\varepsilon = \frac{Q}{4\pi\epsilon r^2} \tag{2}$$

Thus, in general force experienced on the test charge Q_t is given by

$$F_r = Q_t\varepsilon \tag{3}$$

Let the velocity with which Q_t moves be ν (m/s) and mass of the electron be m, then from Newtons Second Law of motion, we have

$$F_r = m \times \text{acceleration} \ = m \times \frac{d\nu}{dt} \tag{4}$$

Equating eqns(3) and (4) we get

$$F_r = Q_t\varepsilon = m \times \frac{d\nu}{dt} \tag{5}$$

(e) <u>Electric Flux Density (D):</u> From the eqn (2) we have $\varepsilon = \frac{Q}{4\pi\epsilon r^2}$

$$= \frac{Q}{4\pi r^2}\frac{1}{\epsilon}$$

$$\Rightarrow \epsilon\varepsilon = \frac{Q}{4\pi r^2} = \frac{\text{Charge}}{\text{Surface area of sphere}}$$

Therefore $\epsilon\varepsilon$ represents the charge present per unit surface area of the sphere, which is defined as the Electric Flux Density (D) measured in C/m^2

$$D = \epsilon\varepsilon \tag{6}$$

(f) <u>Potential (V):</u> The potential V (in Volts) *of point* B with respect to point A is the work done *against the field* in taking a unit positive charge from A to B. So we have

$$V = -\int_A^B \varepsilon dx \tag{7}$$

Differentiating both sides we get

$$\varepsilon = -\frac{dV}{dx} \tag{8}$$

Note:The minus sign shows that the electric field is directed from the region of higher potential to the region of lower potential.

(g) Potential Energy(U): This equals to the potential multiplied by the charge under consideration, or

$$U = qV \tag{9}$$

(h) Law of Conservation of Energy: States that total energy W, which equals the sum of potential energy U and the kinetic energy $\frac{1}{2}m\nu^2$, remains constant. Thus we have

$$W = U + \frac{1}{2}m\nu^2 = qV + \frac{1}{2}m\nu^2 \tag{10}$$

(i) The unit of Energy: The unit of work or energy called electron volt (eV) is defined as

$$1eV = 1.6 \times 10^{-16} \tag{11}$$

The electron volt arises from the fact that if an electron falls through a potential of one volt, its kinetic energy will increase by the decrease in potential energy to maintain the total energy constant

$$U = qV = (1.6 \times 10^{-16})(1) = 1eV$$

2. **Equations for Three Dimensional case:** Let the electric field vector be $\vec{E} = E_x a_x + E_y a_y + E_z a_z$, then the relation between the Electric field vector and potential is given by,

$$\vec{E} = -\nabla.V \tag{12}$$

where V=potential (scalar) and $\nabla = \left(\frac{\partial}{\partial x}a_x + \frac{\partial}{\partial y}a_y + \frac{\partial}{\partial z}a_z\right)$

$$\vec{E} = -\left(\frac{\partial V}{\partial x}a_x + \frac{\partial V}{\partial y}a_y + \frac{\partial V}{\partial z}a_z\right) \tag{13}$$

3. **Gauss Law:** The electric flux through any closed surface equals the charge enclosed. Therefore we have

$$Q = \int_S D_s ds = \int_v \rho_v dv \tag{14}$$

where ρ_v is volume charge density. Thus we have

$$\nabla.\vec{D} = \rho_v \tag{15}$$

substituting eqn.(6) in (15) we get,

$$\nabla.(\epsilon\vec{E}) = \rho_v$$

$$\Rightarrow \epsilon\nabla.\vec{E} = \rho_v$$

in deriving the above equation we have assumed that the ϵ is a constant and hence independent of ∇. Therefore we get

$$\nabla.\vec{E} = \frac{\rho_v}{\epsilon}$$

Now substituting eqn.(12) we get

$$-\nabla(\nabla.V) = \frac{\rho_v}{\epsilon}$$

$$\Rightarrow \nabla^2.V = -\frac{\rho_v}{\epsilon} \tag{16}$$

This is called Poisson Equation given by,

$$\frac{\partial^2 V}{\partial x^2}a_x + \frac{\partial^2 V}{\partial y^2}a_y + \frac{\partial^2 V}{\partial z^2}a_z = -\frac{\rho_v}{\epsilon}$$

For one dimension case the above reduces to

$$\frac{d^2 V}{dx^2} = -\frac{\rho_v}{\epsilon} \tag{17}$$

4. **Nature of Atom:** The force of attraction between the nucleus and the electrons of the atom is given by $\frac{q^2}{4\pi\epsilon_0 r^2}$. This fore must be equal to the product of mass and acceleration towards the nucleus. (from newtons second law). $a = \frac{\nu}{t}$ but $\nu = \frac{r}{t}$, hence $a = \frac{\nu^2}{r}$. Therefore the force of attraction $F_a = \frac{m\nu^2}{r}$. Thus

$$\frac{q^2}{4\pi\epsilon_0 r^2} = F_a = \frac{m\nu^2}{r} \tag{18}$$

From the eqn.(10) and using the relation $V = -E.r$ which is potential of electrons in the electric field E at distance r from the nucleus is

$$= -r.\left(\frac{q}{4\pi\epsilon_0 r^2}\right) = -\frac{q}{4\pi\epsilon_0 r}$$

Hence the equation for the total energy is given by

$$W = \frac{1}{2}m\nu^2 + q\left(-\frac{q}{4\pi\epsilon_0 r}\right)$$

$$\Rightarrow W = \frac{1}{2}m\nu^2 - \frac{q^2}{4\pi\epsilon_0 r} \tag{19}$$

Q: From the eqns.(18) and (19) derive the relation between the radius and the energy of the electron given by

$$W = \frac{-q^2}{8\pi\epsilon_0 r}$$

A : From eqn.(18) we have

$$F_a = \frac{q^2}{4\pi\epsilon_0 r^2} = \frac{m\nu^2}{r}$$

$$\Rightarrow \frac{q^2}{8\pi\epsilon_0 r} = \frac{1}{2}m\nu^2$$

substituting this in eqn.(19) we get

$$W = \frac{q^2}{8\pi\epsilon_0 r} - \frac{q^2}{4\pi\epsilon_0 r}$$

$$\Rightarrow W = \frac{-q^2}{8\pi\epsilon_0 r} \tag{20}$$

5. **Bohr's Atom:**

   (a) Not all energies as given by classical mechanics are possible, but the atom can possess only certain discrete energies. While in states corresponding to these discrete energies, the electron does not emit radiation, and the electron is said to be in a *stationary*, or *non-radiating* state.

   (b) In a transition from one stationary state corresponding to a definite energy W_2 to another stationary state, with an associated energy W_1, radiation will be emitted. The frequency of this radiant energy is given by

   $$f = \frac{W_2 - W_1}{h} \tag{21}$$

   where h is the Plank's constant in J-s $= 6.626 \times 10^{-34}\ J - s$, where W_1, W_2 are energy in Joules and f is the frequency in cycles/sec = Hertz

(c) A stationary state is determined by the condition that the angular momentum of the electron in this state is quantized and must be an integral multiple of $h/2\pi$. Thus

$$m\nu r = \frac{nh}{2\pi} \tag{22}$$

Q: Derive

$$W_n = \frac{-mq^4}{8h^2\epsilon^2}\frac{1}{n^2}$$

using the eqns.(18), (20) and (22)

A: From eqn.(18) we have

$$\frac{q^2}{4\pi\epsilon_0 r^2} = \frac{m\nu^2}{r}$$

and from (22) we have

$$\nu = \frac{nh}{2\pi}\frac{1}{mr}$$

substituting this in the above eqn, we get

$$\frac{q^2}{4\pi\epsilon_0 r^2} = \frac{m}{r}\left(\frac{nh}{2\pi}\frac{1}{mr}\right)^2$$

$$\Rightarrow r = \frac{n^2h^2\epsilon_0}{\pi m q^2} \tag{23}$$

Now substituting r in eqn.(20), we get

$$W = \frac{-q^2}{8\pi\epsilon_0\left(\frac{n^2h^2\epsilon_0}{\pi m q^2}\right)}$$

Thus the equation for the discrete energies is given by

$$W \equiv W_n = \frac{-mq^4}{8h^2\epsilon_0^2 n^2} \tag{24}$$

Q: From the Bohr's theory we have $f = \frac{W_2 - W_1}{h}$. Using this derive

$$\lambda = \frac{12,400}{E_2 - E_1}$$

where λ is the wavelength in $A^o = 10^{-10}$m

A: $W_2 = qE_2$, $W_1 = qE_1$ where $q = 1.6 \times 10^{-19}$, thus

$$f = \frac{e[E_2 - E_1]}{h}$$
$$= \frac{[E_2 - E_1]}{h/e}$$
$$= \frac{[E_2 - E_1]}{4.14125 \times 10^{-15}}$$

we know that $c = f\lambda$, where c= velocity of the light=3×10^8 m/s. Thus we have

$$f = \frac{[E_2 - E_1]}{4.14125 \times 10^{-15}} = \frac{c}{\lambda}$$
$$\Rightarrow \lambda = \frac{1.242375 \times 10^{-6}}{E_2 - E_1}$$
$$\Rightarrow \lambda = \frac{12,400}{E_2 - E_1}$$

where λ is in A^o

6. **Insulator, Semiconductors and Metals :**

   (a) **Insulator:** A very poor conductor of electricity is on insulator. The energy band structure is shown in figure. For example Diamond (carbon) crystal the region containing no quantum states (i.e. prohibited bond) is several electron volts. $E_g \approx 6$eV. This large forbidden bond separates the filled valence region from vacant conduction bond. The energy which is supplied to the electron from an applied field is too small to carry the particle from the filled to the valance or partially filled bond. Since the electron cannot acquire sufficient applied energy, conduction is impossible and hence diamond is an insulator.

   (b) **Semiconductor:** It is the substance for which the width of forbidden energy region is relatively small ($\sim$ eV) is called Semiconductor. Example graphite (crystalline from of carbon but has as crystal symmetry which is different from diamond) has small value of E_g and it is said to be a semiconductor. Example: Si and Ge with $E_g = 0.785$; $E_g = 1.21$ eV respectively at 0^oK. Hence these materials are insulators at low temperatures. However the conductivity increases with increase in temperature.

   (c) **Metal:** A solid which contains a party filled bond structure is called a metal. Under the influence of an applied electric field the electrons may require addition energy and move into higher states. Figure-1 shows the band structure in the three types of materials:

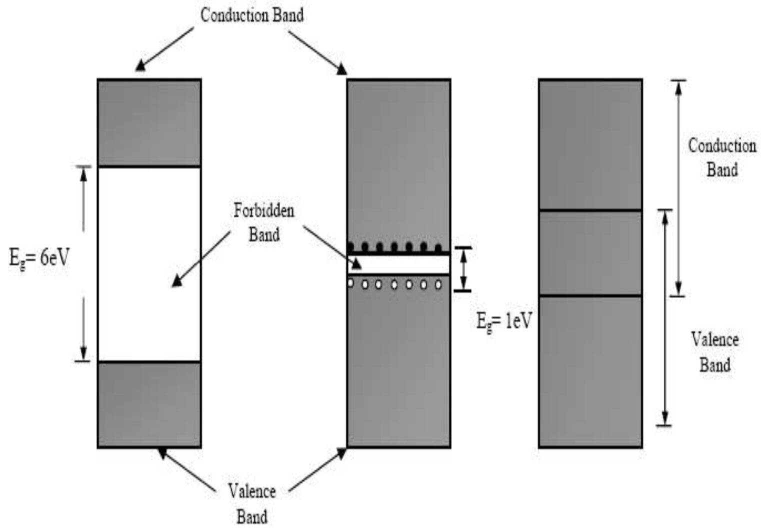

Figure 1: Energy band Structure of (a) Insulator (b) a semiconductor and (c) Metal

7. **Conduction in Metals:** Figure-2 shows a two-dimensional picture of the charge distribution with in the metal. Each immobile ions (nucleus) is associated with valence electrons which are easily broken. Hence these electrons cannot be said to belong to any particular atom. They completely lost their individuality and can wander freely about from atom-to-atom in the metal. These free electrons move in random and their direction of flight being changed at each collision. This collision can be with immobile ions or an electron moving in the opposite direction. However as their motion is random, and also their is no particular orientation of the charge, the net drift of the charge is considered to be zero. Hence the net current is said to be zero.

   Let us now see the situation when constant electric field (ε) is applied. As a result of this electrostatic force, the electrons will be accelerated and the velocity would increase indefinitely with time, only if there were no collisions. However at each inelastic collision, an electron losses its energy, and a steady state condition is reached where a finite value of *drift speed* v is attained. The acceleration is given by

$$F = ma = q\varepsilon$$

$$\Rightarrow a = \frac{q\varepsilon}{m}$$

$$\Rightarrow \left(\frac{v}{t}\right) = \frac{q\varepsilon}{m}$$

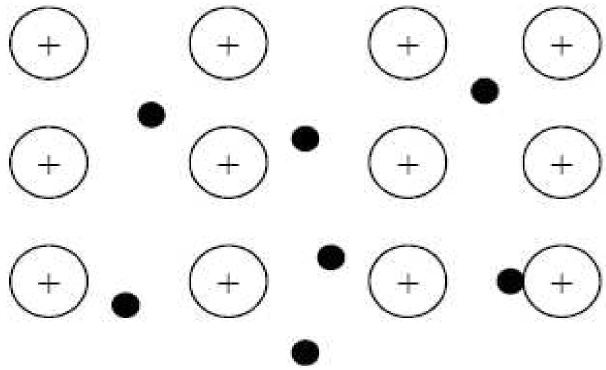

Figure 2: A two dimensional array of ions and free electrons

$$\Rightarrow \upsilon = \left(\frac{qt}{m}\right)\varepsilon$$

The value in the parenthesis of the above equation is called *mobility* of the electrons measure in meters/Volt-sec and denoted by μ. Thus the relation between drift velocity and electric field is given by

$$\upsilon = \mu\varepsilon \tag{25}$$

8. **Derivation of Ohms Law:** If N electrons are contained in a length L of conductor (figure-3), and if it takes a time T second to travel a distance of L meters in the conductor, the total number of electrons passing through any cross section of wire in unit time is N/T. Thus the total charge per second passing any area, which, by definition, is the current in amperes, is

$$I = \frac{Nq}{T} = \frac{Nq}{T}\left(\frac{L}{L}\right)$$
$$= \left(\frac{Nq}{L}\right)\left(\frac{L}{T}\right)$$
$$\Rightarrow I = \left(\frac{Nq}{L}\right)\upsilon \tag{26}$$

By the definition of current density (measured in amperes/m^2), denoted by the symbol J, is the current per unit area of conducting medium. Thus we have

$$J = \frac{I}{A} \tag{27}$$

By substituting eqn.(26) in the above equation, it becomes

$$J = \frac{Nqv}{LA} = \left(\frac{N}{LA}\right) qv \tag{28}$$

The term in the parenthesis of the above equation is called concentration of electrons per unit volume denoted by n and measured in m^{-3}. Thus

$$n = \frac{N}{LA} \tag{29}$$

Also substituting eqn.(25) and eqn.(29) in the eqn.(27) we have

$$J = nq(\mu\varepsilon) = (nq\mu)\varepsilon \tag{30}$$

the value $nq \equiv \rho$ is called charge density, in Coulombs per cubic meter, and $nq\mu = \sigma$ is defined as *conductivity* of metal in $(\Omega - m)^{-1}$. Thus we have

$$J = \sigma\varepsilon \tag{31}$$

This equation is called point form of Ohms Law. The most well know result of the this expression is $V = IR$ can be easily derived. This derivation is left as an exercise to the reader.

Exercise : Derive $V = IR$ where $R = \frac{\rho_s L}{A}$. ρ_s is called the resistivity, which is reciprocal of conductivity (σ).

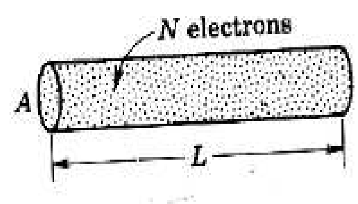

Figure 3: Pertaining to derivation of Ohms Law