Estimating Blocking Probability based on Network Layer and Physical Layer Constraints for High Speed Optical Networks

Balagangadhar B.Gangaya[1], Srinivas Talabathula[1] and Papannareddy Rajappa[2]

[1] Department of Electrical Communication Engineering, Indian Institute of Science, India
[2] College of Engineering and Technology, Purdue University North Central, Indiana, USA
Email: bgbala@ece.iisc.ernet.in

1. INTRODUCTION

Wavelength Division Multiplexing (WDM) is a promising technology for high speed communication. It uses wavelength routing scheme to achieve throughputs of the order of Gb/s to Tb/s. This is because wavelength routed all-optical networks offer wavelength reuse and hence overcome the electronic bottleneck. Lightpaths (LP's) are basic communication channels for communication in wavelength routed optical network. These are analogous to circuits in the traditional circuit switched networks. The requests for the lightpaths arrive at random and are assigned a free wavelength (if available) on each link of the path. A common metric of performance in circuit-switched networks is the call blocking probability, which is defined as the probability that the call cannot be accepted. In telephony networks these call set up and tear down very dynamically. Unlike in all-optical networks these LPs remain for a longer duration. However, blocking probability is still a reasonable metric for optical networks. This is because with the increasing traffic these lightpaths are being leased for varying time durations. Hence a wavelength is increasingly viewed as the circuit [1]. We define this blocking probability as the resource blocking or network layer blocking probability [2]. If the nodes are equipped with wavelength conversion then the lightpaths can be assigned on different wavelengths on the links of the route. This reduces the blocking probability and the optical network behaves as the conventional circuit switched network [3]. However these converters are still at the research state, and increase the cost of the network if implemented. On the other hand, if nodes cannot have wavelength conversion, then the network is called all-optical network, and the network suffers from the *wavelength continuity constraint* [1]. In the high speed networks where the data rates are of the order of Gb/s, there can be many impairments (including the non-linearity) in the physical fibre link which effect the quality of the signal. Hence as the signal goes from the source to the destination signal will degrade drastically and might become unacceptable at the destination end. In this case *bit-error rate* (BER) is the chosen metric. This results in another type of blocking called *physical layer blocking*. There are increasingly many algorithms which have been implemented for taking physical layer issues into the routing algorithms [4] [5] [6]. These algorithms are called *QoS aware routing algorithms*. However these algorithms are not analytically tractable, and hence there is a need to obtain an analytical expressions for estimate blocking probabilities.

With this goal in mind we present an analytical model to estimate the blocking probability based on the network layer and physical layer constraints by modifying the algorithm presented in [4] [6]. This paper is organized in the following way. In Section-2 we outline the assumptions used the assumptions used in the paper. We present the modified algorithm in section-3. With the help of the theorem, we arrive at analytical expressions for the blocking probabilities in section-4 and finally conclude in section 5.

2. ASSUMPTIONS USED IN THE MODEL

In this section we state few assumptions about the network, traffic and physical fiber media for estimating the blocking probability of the optical network without wavelength changers (this makes the network under consideration to be an *all-optical network*)

1. Calls for a node pair arrive according to the Poisson process with rate λ, and the duration of the call is exponentially distributed with unit mean.
2. A call can be accommodated on the route only if the same wavelength is available on all the links of the route. That is there is no wavelength conversion.
3. The wavelength is chosen randomly from the set of free wavelengths. This assumption makes the all the wavelengths identical.

4. BER is independent of the wavelength. This assumption can be justified only when the set of wavelengths are close to each other and hence there will not be any notable difference in the BER.
5. Each link has the same set of wavelengths.

3. SYSTEM MODEL

Fig.1 shows the algorithm used. Here the call arrivals are Poisson with exponential call holding time. Each call is assigned an auxiliary wavelength and routed along the shortest path. Now along the shortest route BER is calculated. If this BER exceeds the threshold BER (BER_{max}) then route is marked as bad quality and is lost. On the other hand if $BER \leq BER_{max}$, then the route is marked as good quality. The estimation of the BER is based on the impairment constraints imposed by the physical fiber media as shown in the Fig.1. We consider the constraints on linear impairments like Chromatic Dispersion (CD), polarization mode dispersion (PMD), amplified spontaneous emission (ASE), and crosstalk (XCS). These constraints ensure that the signal quality is below certain threshold and thus marking the LP as the good quality. Now the auxiliary wavelength is released and new free wavelength available is assigned. Blocking probability (P_b) in the BER module is defined as the ratio number of blocked calls to the total number of calls arrived [4]. In this paper we focus on the arrival process to the network layer module and prove that the arrival rate is dependent on P_b . Fig.2 shows the graph for power versus node. This graph is similar to the one obtained in [4]. As we see from the graph that the BER degrades at each node and if the BER is greater than the threshold BER the call is rejected. Note that we have only considered the XCS an ASE in the graph.

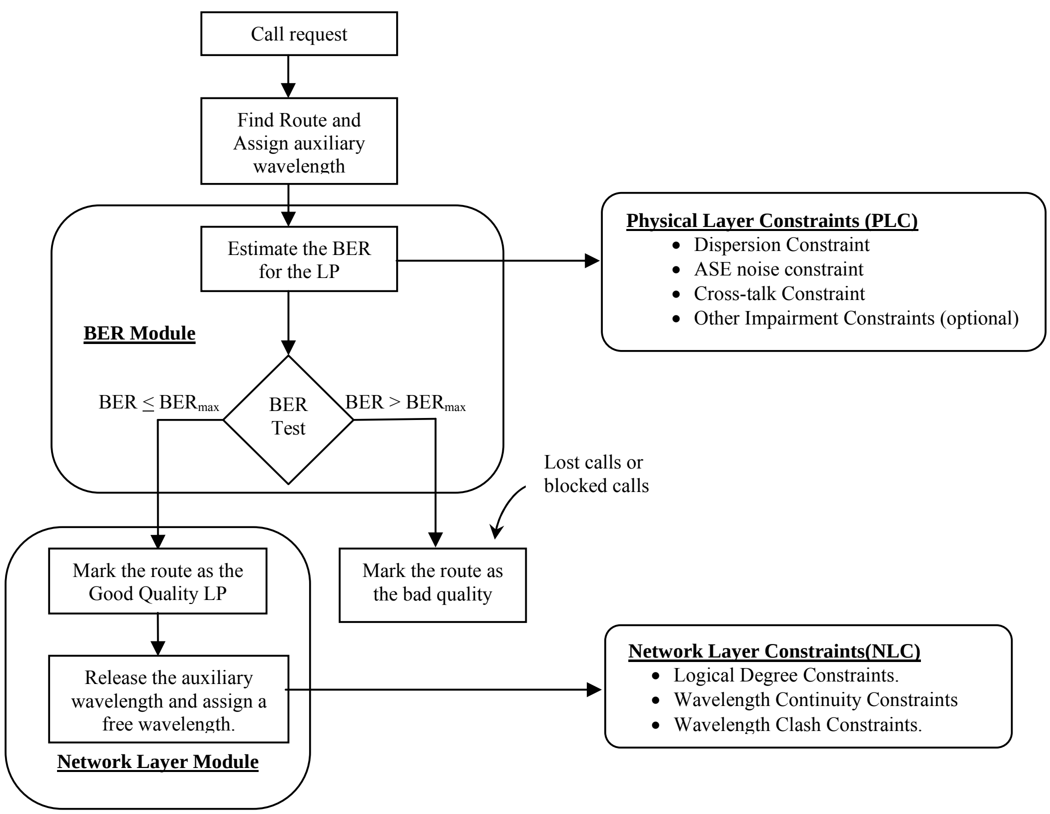

Figure-1: QoS Aware Routing Algorithm used in the proposed model.

3.1 Analytical Model for the Network Layer Module

Let $N(t)$ be the Poisson arrival process with rate λ and denote its jump instants by $\{T_1, T_2, T_3, T_4, \ldots\ldots\}$. These arrival points are indicated with the dark circles as shown in the Fig.3. Consider independent Bernoulli process [7] $Z_k, k \geq 1$; such that Z_k are independent and identically distributed random variables with

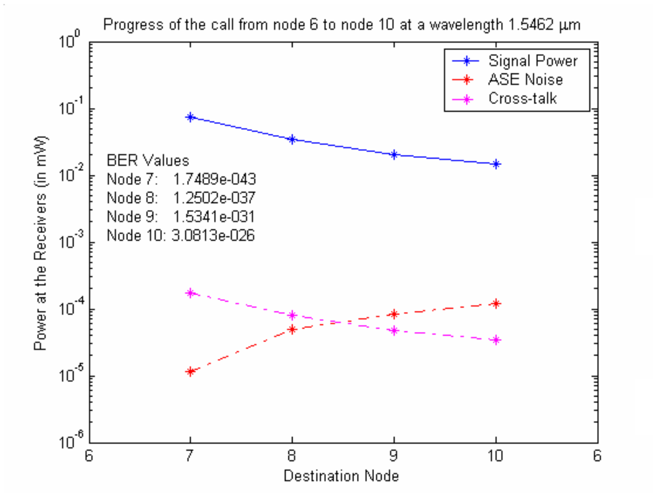

Figure-2: Degradation of the signal from source to the destination node in the ring network and the BER values at each node

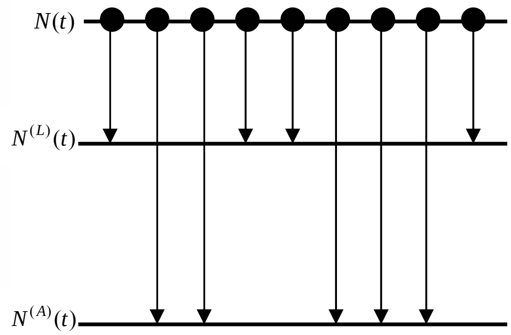

Figure-2: Modeling of the arrival process to the network layer, with Poisson Splitting.

$$Z_k = \begin{cases} 1 & w.p \;\; p \equiv P_b \\ 0 & w.p \;\; 1-p = 1-P_b \end{cases} \tag{1}$$

Z_k can be viewed as the process in which the calls are tagged with probability (w.p) P_b that they do not meet the BER constraint. Now we define new point process $N^{(L)}(t)$ and $N^{(A)}(t)$ as follows: each point $T_k, 1 \le k \le N(t)$, is a point of $N^{(L)}(t)$ if Z_k =1, else T_k is a point of $N^{(A)}(t)$. $N^{(L)}(t)$ represents the process for the lost calls due to the BER constraint with probability P_b and $N^{(A)}(t)$ represents the process for accepted calls which guarantee BER with probability (1- P_b). Thus each point of the Poisson process $N(t)$ is assigned to $N^{(L)}(t)$ or $N^{(A)}(t)$ as shown in the Fig.3, with probability P_b or (1- P_b) respectively and assignment is independent of across the points of $N(t)$. Z_k is used to split the arrival process $N(t)$ into two arrival process. This kind of splitting is called *Bernoulli splitting.*

Theorem: $N^{(L)}(t)$ and $N^{(A)}(t)$ are independent Poisson process with rates λP_b and $\lambda(1-P_b)$, respectively.

Remark: The hypothesis of Bernoulli sampling is crucial. That means there is no dependence between the selection points of $N(t)$. This independent selection is valid in our case since P_b is independent of arrival process $N(t)$.

Proof: Consider the following probability $P(N^{(L)}(t_2) - N^{(L)}(t_1) = k_1, N^{(A)}(t_2) - N^{(A)}(t_1) = k_2)$ where $t_2 > t_1$, we can write,

$$\begin{aligned} P(N^{(L)}(t_2) - N^{(L)}(t_1) = k_1, N^{(A)}(t_2) - N^{(A)}(t_1) = k_2) &= P(N(t_2) - N(t_1) = k_1 + k_2) \times \frac{(k_1+k_2)!}{k_1! k_2!} (P_b)^{k_1} (1-P_b)^{k_2} \\ &= \frac{(\lambda(t_2-t_1))^{k_1+k_2} e^{-\lambda(t_2-t_1)}}{(k_1+k_2)!} \frac{(k_1+k_2)!}{k_1! k_2!} (P_b)^{k_1} (1-P_b)^{k_2} \\ &= \frac{(\lambda P_b (t_2-t_1))^{k_1} e^{-\lambda P_b (t_2-t_1)}}{k_1!} \times \frac{(\lambda(1-P_b)(t_2-t_1))^{k_1} e^{-\lambda(1-P_b)(t_2-t_1)}}{k_2!} \\ &= P(N^{(L)}(t_2) - N^{(L)}(t_1) = k_1) P(N^{(A)}(t_2) - N^{(A)}(t_1) = k_2) \end{aligned}$$

Thus $N^{(L)}(t)$ and $N^{(A)}(t)$ are said to Poisson arrival process with rates λP_b and $\lambda(1-P_b)$ respectively and also $N(t) = N^{(L)}(t) + N^{(A)}(t)$

4. DISCUSSION

Thus from the theorem, we see that the arrival process to the network-layer module follows the Poisson process with new arrival rate $\lambda(1-P_b)$. Let $P_b^{(N)}$ is the network layer blocking probability. Let X_R be the random variable representing the number of idle wavelengths on the route *R*. If the route consists of only single link then we have X_j. Let $R = \{1,2,3,4.........J\}$ denote the route for end-to-end traffic. Then $P_b^{(N)} = P[X_R = 0]$, thus from the [8] we get

$$P_b^{(N)} = \sum_{\mathbf{m}} p_0(m) \prod_{j=1}^{J} P[X_j = m_j] \quad (2)$$

where **m**= ($m_1, m_2, m_3..........m_J$). This relation is obtained by assuming that { X_j } are independent. In the Eqn.(2) we have used the notation,

$$p_n(\mathbf{x}) = P[X_R = n \mid X_1 = x_1, ...X_N = x_N] \quad (3)$$

where $R = \{1,........N\}$ is any route consisting of *N* links, $N \geq 2$, and $\mathbf{x} = (x_1, x_2, ...x_N)$. The other term in the Eqn. (2) is obtained from the solution of the *Erlang-B* [9] formula as

$$P[X_j = m_j] = \frac{\left(\lambda(1-P_b)\right)^{C-m_j}}{(C-m_j)!} \left(\sum_{k=0}^{C} \frac{\lambda^k}{k!} \right)^{-1} \quad (4)$$

5. CONCLUSION

We have presented a model for estimating the blocking probabilities in the QoS aware routing algorithms for estimating blocking probabilities based on the network and physical layer constraints. In addition, we have shown that the network layer blocking probability ($P_b^{(N)}$) is dependent on the physical layer blocking (P_b) as given by equation Eq. (2). Evaluating BER for every arrival helps in marking the routes as good or bad quality. Hence if network relaxes the BER threshold (BER_{max}) later, this marking helps to reject the calls without evaluating BER all over again. If P_b approaches to zero, then arrival rate to the network layer equals the actual arrival rate. This happens only when we place no restriction on the QoS.

6. REFERENCE

[1] A.Sridharan, Kumar.N.Sivarajan, "Blocking in All-Optical Networks", *IEEE/ACM Transactions on Networking*, **vol.12**, No.2, pp.384-397, April 2004.

[2] B.Balagangadhar, T.Srinivas, Rajappa Papannareddy, "Constraint based routing for wavelength routed optical networks", **Proc. ICOL-2005**.

[3] S.Subramaniam, M.Azizoglu, A.K. Somani, "All optical networks with sparse wavelength conversion", *IEEE/ACM Trans. Netw.,* vol.4, pp.544-557, August.1996

[4] B.Ramamurthy, D.Datta, H.Feng, J.P.Heritage, and B.Mukherjee, "Impact of transmission impairments on the teletraffic performance of wavelength routed optical networks," *IEEE/OSA Journal of Lightwave Technology*, **vol.17,** no.10**,** pp.1713-1723 (1999).

[5] T.Deng, and S.Subramaniam, "QoS-friendly wavelength assignment in dynamic wavelength routed optical networks," *Photonics Network Communications*, **vol.10,** no.1, July 2005, pp.5-22.

[6] Y.Haung, J.P.Heritage, and B.Mukherjee, "Connection provisioning with transmission impairment consideration in optical WDM network with high-speed channels", *IEEE/OSA Journal of Lightwave Technology*, **vol.23,** no.3**,** pp.982-993 (2005).

[7] Ronald W. Wolff, *Stochastic modelling and the theory of queues*. New Jersey: Prentice Hall, 1989.

[8] A.Biram, "Computing approximate blocking probabilities for a class of all-optical networks," IEEE Journal Selected Area of Communications, **vol.14**, pp.852-857, June 1996

[9] D.Bertsekas, R.Gallager, *Data Networks, 2nd Edition*, Prentice-Hall, India, 1992.