Complete any two of the following three questions.

P.1a
Suppose two equally probable 1-dimensional densities are of the form

\[ p(x | \omega_i) \propto \exp(-\frac{|x - a_i|}{b_i}) \text{ for } i = 1, 2 \text{ and } b_i > 0 \]

(a) Write an analytic expression for each density function. Namely, you have to normalize \( a_i \) and \( b_i \) for each function.
(b) Write the likelihood ratio \( \frac{p(x | \omega_1)}{p(x | \omega_2)} \).
(c) Sketch a graph of the likelihood ratio for the case \( a_1 = 1, b_1 = 2, a_2 = 0, b_2 = 1 \).

P.1b
Under the Bayesian decision rule, the classification error is given by

\[ P(\text{error}) = \int P(\text{error} | x)p(x)dx = \int \min[P(\omega_1 | x), P(\omega_2 | x)]p(x)dx \]

Show that for arbitrary density functions, an upper bound of the classification error can be found by replacing \( \min[P(\omega_1 | x), P(\omega_2 | x)] \) with \( 2P(\omega_1 | x)P(\omega_2 | x) \) and a lower bound can be found by replacing \( \min[P(\omega_1 | x), P(\omega_2 | x)] \) with \( P(\omega_1 | x)P(\omega_2 | x) \).

P.3 (Matlab Exercise)
(a) Write a function to calculate the discriminant function of the following form for a given mean vector and a covariance matrix.

\[ g_i(x) = -\frac{1}{2}(x - \mu_i)'\Sigma_i^{-1}(x - \mu_i) - \frac{d}{2}\ln 2\pi - \frac{1}{2}\ln |\Sigma_i| + \ln P(\omega_i) \]

for a given mean vector and a covariance matrix.
(b) Write a function to calculate the Mahalanobis distance between an arbitrary point \( x \) and the mean, \( \mu \), of a Gaussian distribution with covariance matrix \( \Sigma \).
(c) Use the data set shown in the table at the beginning of the computer exercises of Chapter 2 (page 79 or 80). Assume each category has Gaussian distribution. Compute the mean and covariance matrix for each category. Assume the prior probabilities are \( P(\omega_1) = 0.8, P(\omega_2) = P(\omega_3) = 0.1 \). Then use your procedures developed in the previous parts (a) and (b) and the minimal error classifier to classify the following test data points: (1,2,1), (5,3,1), (0,0,0), and (1,0,0).