Last lecture:
- Derivative operators: Laplacian, Unsharp Masking

This lecture:
- Image Transform
- Readings: G&W Chap 4, Jain 5.1-5.6, 5.11
Circular Convolution

FIGURE 4.26 Left: convolution of two discrete functions. Right: convolution of the same functions, taking into account the implied periodicity of the DFT. Note in (i) how data from adjacent periods corrupt the result of convolution.

Zero padding

- Convolution without padding
- with padding
- Extended size = A+B-1
Assume original shifting (i.e., (-1)^x+y) has been done.
- Note the blurring and ringing effect

FIGURE 4.12  (a) Original image. (b)-(f) Results of ideal lowpass filtering with cutoff frequencies set at radii values of 5, 15, 30, 80, and 250, as shown in Fig. 4.11(b). The power removed by these filters was 8, 5, 4, 3, 2, and 0.5% of the total, respectively.

Tradeoff between blur and ring

FIGURE 4.2  (a) A discrete function of M points and (b) its Fourier spectrum. (c) A discrete function with twice the number of nonzero points, and (d) its Fourier spectrum.
**Butterworth LPF**

\[ H(u, v) = \frac{1}{(1 + D(u, v)/D_0)^{2n}} \]

- \( D_0 \) controls the cutoff frequency

![Butterworth LPF Diagram](image)

**Figure 4.14** (a) Perspective plot of a Butterworth lowpass filter transfer function. (b) Filter displayed as an image. (c) Filter radius cross sections of orders 1 through 4.

- Butterworth LPF
- Order = 2
- No ringing

![Butterworth Filter Images](image)
**Effect of the order**

- When order=20, close to Ideal LPF

![Image of filters and frequency responses]

**Gaussian LPF**

\[ H(u, v) = e^{-D^2(u,v)/2D_0^2} \]

![Image of Gaussian LPF transfer function and frequency response]

**FIGURE 4.16** (a) - (c) Spatial representation of RLPFs of order 1, 2, 5, and 20, and corresponding grey-level profiles through the center of the filters (all filters have a cutoff frequency of 3). Note that ringing increases as a function of filter order.

**FIGURE 4.17** (a) Perspective plot of a GLLPF transfer function. (b) Filter displayed as an image. (c) Filter radial cross sections for various values of \( D_0 \).
- Gaussian
- No ringing effect in spatial domain

**Text Smoothing**

**FIGURE 4.19**
(a) Sample text of poor resolution (note broken characters in magnified view).
(b) Result of filtering with a GLPF (broken character segments were joined).

Historically, certain computer programs were written using only two digits rather than four to define the applicable year. Accordingly, the company's software may recognize a date using "00" as 1900 rather than the year 2000.