Wavelet & Subband Coding.

Reading: Chap 7.

- Transform

\[ X(z) = \sum_{n=-\infty}^{\infty} x(n) z^{-n} \]

\[ z = e^{j\omega} \]

\[ X(\omega) = \frac{1}{2} \left[ X(\omega/2) + X(-\omega/2) \right] \]

\[ z = e^{j\omega} \]

\[ X_{\text{down}}(\omega) = \frac{1}{2} \left[ X(\omega/2) + X(\omega/2 + \pi) \right] \]

\[ z = e^{j\omega} \]

\[ X_{\text{down}}(z) = \frac{1}{z} \left[ X(z^{1/2}) + X(-z^{1/2}) \right] \]

\[ z = e^{j\omega/2} \]

\[ X_{\text{down}}(z) = e^{j\omega/2} \]

\[ z = e^{j\omega/2} \]

\[ X_{\text{down}}(z) = e^{j(\omega/2+\pi)} \]

Analysis

Synthesis

Eleven Trans.
\[
X(n) \rightarrow \uparrow 2 \rightarrow X^{up}(n) = \begin{cases} 
5x(n/2) & n = 0, 2, 4 \\
0 & n = \text{odd} 
\end{cases}
\]

\[
X^{up}(z) = X(z^2)
\]

\[
X^{up}(\omega) = X(2\omega)
\]

Aliasing Copy

Modulated

High Freq.
\[ X \rightarrow \Box \downarrow \rightarrow \Box \rightarrow \hat{X} \]

\[ \hat{X}(z) = \frac{1}{z} \left[ X(z) + X(-z) \right] \]

\[ \hat{X}(\omega) = \frac{1}{2} \left[ X(\omega) + X(\omega^*) \right] \]

\[ \hat{X}(z) = \frac{1}{z} G_0(z) \left[ H_0(z) X(z) + H_0(-z) \overline{X(-z)} \right] + \frac{1}{z} G_1(z) \left[ H_1(z) X(z) - H_1(-z) \overline{X(-z)} \right] \]

\[ = \frac{1}{z} \left[ H_0(z) G_0(z) + H_1(z) G_1(z) \right] X(z) \]

\[ + \frac{1}{z} \left[ H_0(-z) G_0(z) + H_1(-z) G_1(z) \right] \overline{X(-z)} \]

\[ = X(z) \]

\[ \sum_{n=0}^{\infty} h(n) z^{-n} \]

**Perfect Reconstruction Conditions:**

\[ h_0(n) = \left\{ \begin{array}{ll} 1 & \text{if } n = 0 \\ 0 & \text{if } n \neq 0 \end{array} \right. \quad h_0(z) = 1 + z^{-1} \]

\[ h_1(n) = \left\{ \begin{array}{ll} 1 & \text{if } n = 1 \\ 0 & \text{if } n \neq 0 \end{array} \right. \quad h_1(z) = 1 - z^{-1} \]

\[ g_0(n) = \left\{ \begin{array}{ll} 1 & \text{if } n = 0 \\ 0 & \text{if } n \neq 0 \end{array} \right. \quad g_0(z) = 1 + z^{-1} \]

\[ g_1(n) = \left\{ \begin{array}{ll} 1 & \text{if } n = 1 \\ 0 & \text{if } n \neq 0 \end{array} \right. \quad g_1(z) = 1 - z^{-1} \]

\[ (1 + z^{-1}) (1 + z^{-1}) + (1 - z^{-1}) (1 - z^{-1}) \]

\[ = 4 \]

13-3
\[ x(n) \rightarrow \left[ \begin{array}{c} h(n) \\end{array} \right] \rightarrow y(n) \]

\[ y(n) = x \oplus h = \sum_{k=0}^{N} x(k) h(n-k) \]

\[ y(n) = \sum_{k=0}^{N} x(k) h(n-k) \]

\[ x(n) = [16 \ 8 \ 9 \ 5 \ 10 \ 4 \ 3 \ 5] \]

\[ x(n) \rightarrow [16 \ 8 \ 9 \ 5 \ 8 \ 4 \ 3 \ 5] \]

\[ A_1: \ 16 \ 8 \ 9 \ 5 \ 10 \ 4 \ 3 \ 5 \]

\[ D_1: \ 8 \ 4 \ 6 \ -2 \]

\[ A_{1,u}^T \left[ \begin{array}{c} 24 \ 14 \ 14 \ 8 \end{array} \right] \]

\[ D_{1,u}^T \left[ \begin{array}{c} 8 \ 4 \ 6 \ -2 \end{array} \right] \]

\[ A_{1,8}^T \left[ \begin{array}{c} 24 \ 24 \ 14 \ 14 \ 14 \ 14 \ 8 \ 8 \end{array} \right] \]

\[ D_{1,8}^T \left[ \begin{array}{c} 8 \ -8 \ 4 \ -4 \ 6 \ -6 \ -2 \ 2 \end{array} \right] \]

\[ 13-4 \]
JPEG-2000: DWT
JPEG: DCT

Use filter to model image manipulation

\[ f(x) \xrightarrow{\text{object}} \text{Motion} \quad \alpha \text{ pixels/sec} \]

\[ \downarrow \text{Camera: shutter opens at time } t = 0 \quad \text{closes at } t = T \]

Image when static \( f(x) \)

\[ \rightarrow \text{Output: Motion-Blur.} \]

\[ f(x) \xrightarrow{?} g(x) : \text{motion blur, image} \]

\[ \hat{f}(x) \xleftarrow{\text{Restored}} \]

Time-domain

\[ g(x) = \int_{t=0}^{T} f(x-\alpha t) dt \]

\[ \text{Fourier-Domain} \]

\[ \int_{0}^{T} g(x) e^{-j\omega x} dx = \int_{0}^{\infty} \left[ \int_{t=0}^{T} f(x-\alpha t) dt \right] e^{-j\omega x} dx \]

\[ \mathcal{F}\{g(x)\} = \mathcal{F}\{f(x)\} \left[ \int_{0}^{T} e^{-j\omega \alpha t} dt \right] \]

\[ G(\omega) = F(\omega) \left[ \int_{0}^{T} e^{-j\omega \alpha t} dt \right] \]

\[ H(\omega) \]

Exercise: 13-5
\[ H(\omega) = \int_{-\infty}^{\infty} f(t) e^{-j\omega t} dt \]

\[ = \frac{g}{\omega} \sin \left( \frac{\omega_0 T}{2} \right) e^{-\frac{i\omega_0}{2}} \]

\[ \sim \frac{\sin}{\omega_0} |H(\omega)| \]

\[ T \uparrow : \text{shutter slow} \]

\[ a \uparrow : \text{blur severe} \]

\[ B \uparrow : \text{blur severe} \]