1 Image Quantization and Quality Measurement

Uniform Quantizor

Denote $u$ as the quantizor input and $v = Q(u)$ as the quantizor output. Then the generalized quantizor can be defined by

$$v = Q(u) = r_i, \quad \text{if } t_i \leq u < t_{i+1}, \quad i = 0, 1, \ldots, N - 1,$$

where $\{t_i\}_i$ are the decision boundaries and $\{r_i\}_i$ are the reconstruction values. For a uniform quantizor, the decision boundaries and the reconstruction values satisfy

$$r_i = \frac{t_i + t_{i+1}}{2},$$

and

$$t_i = i\Delta, \quad i = 0, 1, 2, \ldots, N - 1,$$

where $\Delta$ is the quantization step size. Using (3), (2) can be further rewritten as

$$r_i = \frac{(2i + 1)}{2}\Delta = (i + \frac{1}{2})\Delta.$$
Moreover, for a given input value $u$, the index of the corresponding quantization region, $i$, in a uniform quantizer can be expressed by

$$i = \left\lfloor \frac{u}{\Delta} \right\rfloor. \quad (5)$$

Inserting $i$ defined in (5) into (4), we then have the output of a uniform quantizer:

$$v = Q(u) = \left( \left\lfloor \frac{u}{\Delta} \right\rfloor + \frac{1}{2} \right) \Delta. \quad (6)$$

Note that for a 3D image, the uniform quantizer given in (6) is applicable for each color channel separately. Suppose each pixel has the depth of $B_R$, $B_G$, and $B_B$ bits in R, G, and B color channels, respectively. Let $n_R$, $n_G$, and $n_B$ be the numbers of quantization steps in R, G, and B color dimensions, respectively. Then the quantization step sizes for R, G, and B color channels in (6) are given by

$$\begin{align*}
\Delta_R &\approx \frac{2^{B_R}}{n_R}, \\
\Delta_G &\approx \frac{2^{B_G}}{n_G}, \quad \Rightarrow \Delta \approx \frac{2^B}{n}.
\end{align*} \quad (7)$$

**SNR of 3D Images**

The signal-to-noise ratio (SNR) of an image in each single color channel is defined by

$$\text{SNR}_l = \frac{P^l_s}{P^l_n} = \frac{\sum_m \sum_n [u(m,n,l) - \bar{u}(l)]^2}{\sum_m \sum_n [u(m,n,l) - v(m,n,l)]^2}, \quad l = R, G, B, \quad (8)$$

where $u(m,n,l)$ and $v(m,n,l)$ denotes the $(m,n)$-th original pixel value and the corresponding quantized pixel value, respectively, in the color channel $l$, $1 \leq m \leq M$ and $1 \leq n \leq N$; $\bar{u}(l) \triangleq \frac{1}{MN} \sum_m \sum_n u(m,n,l)$ is defined as the average pixel value of the image in the color channel $l$. Then the overall SNR of a 3D image is given by

$$\text{SNR} = \frac{\sum_l P^l_s}{\sum_l P^l_n} = \frac{\sum_l \sum_m \sum_n [u(m,n,l) - \bar{u}(l)]^2}{\sum_l \sum_m \sum_n [u(m,n,l) - v(m,n,l)]^2}. \quad (9)$$
Simulation Results

Using “lena.bmp” as the input image where $B_R = B_G = B_B = 8$bits, we implement the uniform quantizor (6) to obtain the quantized image and calculate the corresponding SNR using (9). Fig. 1 and Fig. 2 show the results of $n_R = n_G = n_B = 4$ and $n_R = n_G = n_B = 6$, respectively. As a comparison, the original “lena.bmp” image is also shown in the figures.

2 HDTV Resolution

First, compute the number of pixels per line by

\[
\frac{16}{9} = \frac{L}{1125} \Rightarrow L = 2000.
\]

Then the overall data rate can be calculated by

\[
(2000 \times 1125) \text{ pixels/frame} \times \frac{60 \text{ frames}}{2} \times 24 \text{ bits/pixel} \times 3600 \text{ sec/hour} \times 2 \text{ hours} \\ \approx 1.166 \times 10^{13} \text{ bits}.
\]
Figure 1: The output image of the uniform quantizer (6): $B_R = B_G = B_B = 8$ bits; $n_R = n_G = n_B = 4$; $\mathbb{SNR} \approx 8.0$ dB.
Figure 2: The output image of the uniform quantizer (6): \( B_R = B_G = B_B = 8 \text{bits}; n_R = n_G = n_B = 6; \overline{\text{SNR}} \approx 11.6 \text{dB}. \)