EE E4830 Digital Image Processing, Spring 2005
Prof. S.-F. Chang

Final Exam
May 12th, 2005 Thursday 4:10 pm – 6:30 pm

Note:
1. Please answer all four problems with clear labels of your answers.

2. Please use the standard blue color exam book only. Only answers written on the exam book will be graded. Remember to write your name clearly on the cover page.

3. Open books and notes. Use of calculator is OK. But computers, cellular phones, and all networked devices are not allowed.

4. Best luck! Have a great summer!
P.1 (Edge Detection) (15%)

Consider a one-dimensional signal that constitutes some edges and variations depicted below. Sketch the responses when the gradient and Laplacian operators are applied to the signal respectively. Exact numerical values are not required. Plot of approximate shapes of the responses will be sufficient.

![Signal with edges](image)

P.2 (Morphological Operations and Image Classification) (30%)

You are asked to design an image processing and recognition system to detect and classify elliptical shapes in images similar to the one shown below. All the blobs in the image are of elliptical shapes and of three different sizes. The average values of the principal axes of the ellipses are (1.3, 0.7), (1.0, 0.5), and (0.75, 0.25). The actual values of the axes vary within 10% about their average values.

Develop an image processing system that is capable of detecting only the complete, non-overlapping ellipses, and then classifying each of these ellipses into one of the three classes described above. Note all the ellipses touching the boundaries or overlapping with others should be rejected.

Show a block diagram with sufficient details about each block to describe your solutions. For your classification module, explain with details the classification algorithm you adopt and the procedures for using the training samples to train your classifiers.

[Hint: Some morphological functions such as those detecting connected components or convex halls might be useful for detecting the non-overlapping ellipses. However, feel free to propose different methods of your own choices.]
P.3 (Invariant Moment Features) (30%)

We have learned the representation of segmented regions using central moments. Note in the lecture we used discrete summation. Here we assume the image is defined over the continuous coordinate space and the summation can be changed to continuous integral.

\[
\mu_{p,q} = \iint (x - \bar{x})^p (y - \bar{y})^q f(x, y) \, dx \, dy
\]

where \( \bar{x} \) and \( \bar{y} \) are the center locations of the region, and \( f(x, y) \) is the binary function indicating the support of the region.

\[
f(x, y) = \begin{cases} 
1, & \text{if } (x, y) \in R \\
0, & \text{if } (y, y) \notin R 
\end{cases}
\]

where \( R \) is the region.

In HW#5, we have shown the normalized central moments, \( \eta_{p,q} \), defined below is invariant to scale change.

\[
\eta_{p,q} = \frac{\mu_{p,q}}{\mu_{0,0}^{p+q+2}}
\]

Here, prove the term, \( \phi = \eta_{20} + \eta_{02} \), is invariant to translation, rotation, and scale change. Terms like this are called invariant moments and have been found useful for object matching.
We have shown in the class how to use Hough Transform to detect lines in an image. As shown in the two figures above, a point \((x,y)\) on a line is mapped to a line, \(b = -xa + y\), in the Hough Transform parameter space. Multiple points on the same line in the image space are mapped to multiple lines in the parameter space. Their intersection point uniquely determines the parameters of the line.

In this problem, we want to extend the method to detection of circles in an image.

(a) First, write down the equation defining a circle with center \((x_0, y_0)\) and radius \(R\) in the image space.

(b) Similar to the case discussed above for line detection, parameters \((x_0, y_0, R)\) defines a circle in the image space and can be represented by a point in the Hough Transform parameter space. Now given a point \((x,y)\) located on a circle in the image space, in the Hough Transform parameter space find the points corresponding to all of the circles passing the point \((x,y)\). What is the shape defined by these points in the Hough Transform parameter space?

(c) Describe the step-by-step procedures that use Hough Transform described above to detect circles in an image.