Fig. (a) The signal $f(t)$ in this case is a triangle pulse $\Delta(t)$ (Fig. S4.3-6) multiplied by $\cos 10t$.

$$f(t) = \Delta \left(\frac{t}{2\pi}\right) \cos 10t$$

Also from Table 4.1 (pair 19) $\Delta(t) \leftrightarrow \pi \text{sinc}^2(\frac{\pi t}{2})$. From the modulation property (4.41), it follows that

$$f(t) = \Delta \left(\frac{t}{2\pi}\right) \cos 10t \leftrightarrow \frac{\pi}{2} \left\{ \text{sinc}^2 \left[\frac{\pi(\omega - 10)}{2}\right] + \text{sinc}^2 \left[\frac{\pi(\omega + 10)}{2}\right] \right\}$$

The Fourier transform in this case is a real function and we need only the amplitude spectrum in this case as shown in Fig. S4.3-6a.

Fig. (b) The signal $f(t)$ here is the same as the signal in Fig. (a) delayed by $2\pi$. From time shifting property, its Fourier transform is the same as in part (a) multiplied by $e^{-j2\pi t}$. Therefore

$$F(\omega) = \frac{\pi}{2} \left\{ \text{sinc}^2 \left[\frac{\pi(\omega - 10)}{2}\right] + \text{sinc}^2 \left[\frac{\pi(\omega + 10)}{2}\right] \right\} e^{-j2\pi \omega}$$

The Fourier transform in this case is the same as that in part (a) multiplied by $e^{-j2\pi t}$. This multiplying factor represents a linear phase spectrum $-2\pi \omega$. Thus we have an amplitude spectrum [same as in part (a)] as well as a linear phase spectrum $\angle F(\omega) = -2\pi \omega$ as shown in Fig. S4.3-6b. The amplitude spectrum in this case is shown in Fig. S4.3-6b.

Note: In the above solution, we first multiplied the triangle pulse $\Delta(t)$ by $\cos 10t$ and then delayed the result by $2\pi$. This means the signal in Fig. (b) is expressed as $\Delta(t - 2\pi) \cos 10(t - 2\pi)$.

We could have interchanged the operation in this particular case, that is, the triangle pulse $\Delta(t - 2\pi)$ is first delayed by $2\pi$ and then the result is multiplied by $\cos 10t$. In this alternate procedure, the signal in Fig. (b) is expressed as $\Delta(t) \cos 10(t - 2\pi)$.

This interchange of operation is permissible here only because the sinusoid $\cos 10t$ executes integral number of cycles in the interval $2\pi$. Because of this both the expressions are equivalent since $\cos 10(t - 2\pi) = \cos 10t$.

Fig. S4.3-6
4.3-7

(b)

\[ F(\omega) = \Delta \left( \frac{\omega + 4}{4} \right) + \Delta \left( \frac{\omega - 4}{4} \right) \]

Also

\[ \frac{1}{\pi} \text{sinc}^2(t) \iff \Delta \left( \frac{\omega}{4} \right) \]

Therefore

\[ f(t) = \frac{2}{\pi} \text{sinc}^2(t) \cos 4t \]

4.3-9 From the frequency convolution property, we obtain

\[ f^2(t) \iff \frac{1}{2\pi} F(\omega) * F(\omega) \]

Because of the width property of the convolution, the width of \( F(\omega) * F(\omega) \) is twice the width of \( F(\omega) \). Repeated application of this argument shows that the bandwidth of \( f^n(t) \) is \( nB \) Hz (\( n \) times the bandwidth of \( f(t) \)).

4.6-4 Recall that

\[ f_2(t) = \frac{1}{2\pi} \int_{-\infty}^{\infty} F_2(\omega) e^{j\omega t} d\omega \quad \text{and} \quad \int_{-\infty}^{\infty} f_1(t) e^{j\omega t} dt = F_1(-\omega) \]

Therefore

\[
\int_{-\infty}^{\infty} f_1(t)f_2(t) \, dt = \frac{1}{2\pi} \int_{-\infty}^{\infty} f_1(t) \left[ \int_{-\infty}^{\infty} F_2(\omega) e^{j\omega t} \, d\omega \right] dt \\
= \frac{1}{2\pi} \int_{-\infty}^{\infty} F_2(\omega) \left[ \int_{-\infty}^{\infty} f_1(t) e^{j\omega t} \, dt \right] d\omega = \frac{1}{2\pi} \int F_1(-\omega) F_2(\omega) \, d\omega
\]

Interchanging the roles of \( f_1(t) \) and \( f_2(t) \) in the above development, we can show that

\[
\int_{-\infty}^{\infty} f_1(t)f_2(t) \, dt = \frac{1}{2\pi} \int_{-\infty}^{\infty} F_1(\omega) F_2(-\omega) \, d\omega
\]
4.6-5 Application of duality property [Eq. (4.31)] to pair 3 (Table 4.1) yields

\[
\frac{2a}{t^2 + a^2} \iff 2\pi e^{-\eta|\omega|}
\]

The signal energy is given by

\[
E_f = \frac{1}{\pi} \int_0^\infty |2\pi e^{-a\omega}|^2 d\omega = 4\pi \int_0^\infty e^{-2a\omega} d\omega = \frac{2\pi}{a}
\]

The energy contained within the band (0 to \(W\)) is

\[
E_W = 4\pi \int_0^W e^{-2a\omega} d\omega = \frac{2\pi}{a} \left[ 1 - e^{-2aW} \right]
\]

If \(E_W = 0.99E_f\), then

\[
e^{-2aW} = 0.01 \implies W = \frac{2.3025}{a} \text{ rad/s} = \frac{0.366}{a} \text{ Hz}
\]
4.7-1 (i) For \( m(t) = \cos 1000t \)

\[
\varphi_{DSB-SC}(t) = m(t) \cos 10,000t = \cos 1000t \cos 10,000t \\
= \frac{1}{2} \left( \cos 9000t + \cos 11,000t \right)
\]

(ii) For \( m(t) = 2 \cos 1000t + \cos 2000t \)

\[
\varphi_{DSB-SC}(t) = m(t) \cos 10,000t = [2 \cos 1000t + \cos 2000t] \cos 10,000t \\
= \cos 9000t + \cos 11,000t + \frac{1}{2} \left( \cos 8000t + \cos 12,000t \right) \\
= \frac{1}{2} \left( \cos 9000t + \cos 8000t \right) + \frac{1}{2} \left( \cos 11,000t + \cos 12,000t \right)
\]

(iii) For \( m(t) = \cos 1000t \cos 3000t \)

\[
\varphi_{DSB-SC}(t) = m(t) \cos 10,000t = \frac{1}{2} \left[ \cos 2000t + \cos 4000t \right] \cos 10,000t \\
= \frac{1}{2} \left[ \cos 8000t + \cos 12,000t \right] + \frac{1}{2} \left[ \cos 6000t + \cos 14,000t \right] \\
= \frac{1}{2} \left[ \cos 8000t + \cos 6000t \right] + \frac{1}{2} \left[ \cos 12,000t + \cos 14,000t \right]
\]

This information is summarized in a table below. Figure S4.7-1 shows various spectra.

\[
\begin{array}{cccc}
(i) & M(\omega) & Modulated signal spectrum & \\
-1000 & 1000 & \omega & \rightarrow & -11,000 & 9,000 & 11,000 \\
\hline
(ii) & 2000 & 0 & 2000 & \omega & \rightarrow & -12,000 & -9,000 & -8,000 & 0 & 8,000 & 12,000 \\
\hline
(iii) & 4000 & 0 & 2000 & \omega & \rightarrow & -4000 & -12,000 & -8,000 & -4,000 & 0 & 8,000 & 12,000 & 14,000 \\
\end{array}
\]

Fig. S4.7-1

<table>
<thead>
<tr>
<th>case</th>
<th>Baseband frequency</th>
<th>DSB frequency</th>
<th>LSB frequency</th>
<th>USB frequency</th>
</tr>
</thead>
<tbody>
<tr>
<td>i</td>
<td>1000</td>
<td>9000 and 11,000</td>
<td>9000</td>
<td>11,000</td>
</tr>
<tr>
<td>ii</td>
<td>1000</td>
<td>9000 and 11,000</td>
<td>9000</td>
<td>11,000</td>
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<td>8000 and 12,000</td>
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<td>12,000</td>
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<tr>
<td>iii</td>
<td>2000</td>
<td>8000 and 12,000</td>
<td>8000</td>
<td>12,000</td>
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<tr>
<td></td>
<td>4000</td>
<td>6000 and 14,000</td>
<td>6000</td>
<td>14,000</td>
</tr>
</tbody>
</table>