3.4-1 Here $T_0 = 2$, so that $\omega_0 = 2\pi/2 = \pi$, and

$$f(t) = a_0 + \sum_{n=1}^{\infty} a_n \cos nt + b_n \sin nt$$

where

$$a_0 = \frac{1}{2} \int_{-1}^{1} t^2 \, dt = \frac{1}{3}, \quad a_n = \frac{2}{\pi} \int_{-1}^{1} t \cos nt \, dt = \frac{4(-1)^n}{\pi n^2}, \quad b_n = \frac{2}{\pi} \int_{-1}^{1} t \sin nt \, dt = 0$$

Therefore

$$f(t) = \frac{1}{3} + \frac{4}{\pi} \sum_{n=1}^{\infty} \frac{(-1)^n}{n^2} \cos nt$$

Figure S3.4-1 shows $f(t) = t^2$ for all $t$ and the corresponding Fourier series representing $f(t)$ over $(-1, 1)$.

3.4-3 (a) $T_0 = 4, \omega_0 = \frac{2\pi}{2} = \frac{\pi}{2}$. Because of even symmetry, all sine terms are zero.

$$f(t) = a_0 + \sum_{n=1}^{\infty} a_n \cos \left(\frac{n\pi}{2} t\right)$$

$a_0 = 0$ (by inspection)

$$a_n = \frac{4}{\pi} \left[ \int_{0}^{1} \cos \left(\frac{n\pi}{2} t\right) \, dt - \int_{1}^{2} \cos \left(\frac{n\pi}{2} t\right) \, dt \right] = \frac{4}{n\pi} \sin \frac{n\pi}{2}$$

Therefore, the Fourier series for $f(t)$ is

$$f(t) = \frac{4}{\pi} \left( \cos \frac{\pi t}{2} - \frac{1}{3} \cos \frac{3\pi t}{2} + \frac{1}{5} \cos \frac{5\pi t}{2} - \frac{1}{7} \cos \frac{7\pi t}{2} + \ldots \right)$$

Here $b_n = 0$, and we allow $C_n$ to take negative values. Figure S3.4-3a shows the plot of $C_n$. 
(d) \( T_0 = \pi, \ \omega_0 = \frac{2}{\pi} \) and \( f(t) = \frac{1}{\pi} t. \)
\( a_0 = 0 \) (by inspection).
\( a_n = 0 \) \((n > 0)\) because of odd symmetry.
\[
b_n = \frac{4}{\pi} \int_0^{\pi/4} \frac{4}{\pi} t \sin 2nt \, dt = \frac{2}{\pi n} \left( \frac{2}{\pi} \sin \frac{\pi n}{2} - \cos \frac{\pi n}{2} \right)
\]
\[
f(t) = \frac{4}{\pi^2} \sin 2t + \frac{1}{\pi} \sin 4t - \frac{4}{9\pi^2} \sin 6t - \frac{1}{2\pi} \sin 8t + \cdots
\]
\[
= \frac{4}{\pi^2} \cos \left(2t - \frac{\pi}{2}\right) + \frac{1}{\pi} \cos \left(4t - \frac{\pi}{2}\right) + \frac{4}{9\pi^2} \cos \left(6t + \frac{\pi}{2}\right) + \frac{1}{\pi} \cos \left(8t + \frac{\pi}{2}\right) + \cdots
\]
Figure S3.4-3d shows the plot of \( C_n \) and \( \theta_n \).

(f) \( T_0 = 6, \ \omega_0 = \pi/3, \ a_0 = 0.5 \) (by inspection). Even symmetry; \( b_n = 0 \).
\[
a_n = \frac{4}{6} \int_0^3 f(t) \cos \frac{n\pi t}{3} \, dt
\]
\[
= \frac{2}{3} \int_0^1 \cos \frac{n\pi t}{3} \, dt + \int_1^2 (2 - t) \cos \frac{n\pi t}{3} \, dt
\]
\[
= \frac{6}{\pi^2 n^2} \left[ \cos \frac{n\pi}{3} - \cos \frac{2n\pi}{3} \right]
\]
\[
f(t) = 0.5 + \frac{6}{\pi^2} \left( \cos \frac{\pi}{3} t - \frac{2}{9} \cos \pi t + \frac{1}{25} \cos \frac{5\pi}{3} t + \frac{1}{49} \cos \frac{7\pi}{3} t + \cdots \right)
\]
Observe that even harmonics vanish. The reason is that if the dc (0.5) is subtracted from \( f(t) \), the resulting function has half-wave symmetry. (See Prob. 3.4-7). Figure S3.4-3f shows the plot of \( C_n \).

Fig. S3.4-3
3.4-8 (a) Here, we need only cosine terms and \( \omega_0 = \frac{\pi}{70} \). Hence, we must construct a pulse such that it is an even function of \( t \), has a value \( t \) over the interval \( 0 \leq t \leq 1 \), and repeats every 4 seconds as shown in Fig. S3.4-8a. We selected the pulse width \( W = 2 \) seconds. But it can be anywhere from 2 to 4, and still satisfy these conditions. Each value of \( W \) results in different series. Yet, all of them converge to \( t \) over 0 to 1, and satisfy the other requirements. Clearly, there are infinite number of Fourier series that will satisfy the given requirements. The present choice yields

\[
f(t) = a_0 + \sum_{n=1}^{\infty} a_n \cos \left( \frac{n \pi}{2} \right) t
\]

By inspection, we find \( a_0 = 1/4 \). Because of symmetry \( b_n = 0 \) and

\[
a_n = \frac{4}{\pi} \int_{0}^{1} t \cos \frac{n \pi}{2} t dt = \frac{4}{n \pi} \left\{ \cos \left( \frac{n \pi}{2} \right) \cos \left( \frac{n \pi}{2} \right) - 1 \right\}
\]

(c) Here, we need both sine and cosine terms and \( \omega_0 = \frac{\pi}{70} \). Hence, we must construct a pulse such that it has no symmetry of any kind, has a value \( t \) over the interval \( 0 \leq t \leq 1 \), and repeats every 4 seconds as shown in Fig. S3.4-8c. As usual, the pulse width can be have any value in the range 1 to 4.

\[
f(t) = a_0 + \sum_{n=1}^{\infty} a_n \cos \left( \frac{n \pi}{2} \right) t + b_n \sin \left( \frac{n \pi}{2} \right) t
\]

By inspection, \( a_0 = 1/8 \) and

\[
a_n = \frac{2}{\pi} \int_{0}^{1} t \cos \frac{n \pi}{2} t dt = \frac{2}{n \pi} \left\{ \sin \left( \frac{n \pi}{2} \right) + \frac{n \pi}{2} \sin \left( \frac{n \pi}{2} \right) - 1 \right\}
\]

\[
b_n = \frac{2}{\pi} \int_{0}^{1} t \sin \frac{n \pi}{2} t dt = \frac{2}{n \pi} \left\{ - \sin \left( \frac{n \pi}{2} \right) + \frac{n \pi}{2} \cos \left( \frac{n \pi}{2} \right) \right\}
\]

3.4-9

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<td>--</td>
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3.5.1
\[ f(t) = D_0 + \sum_{n=-\infty}^{\infty} D_n e^{jnt}, \] 
where, by inspection \( D_0 = 0.5 \)

\[ D_n = \frac{1}{2\pi} \int_{-\pi}^{\pi} f(t) e^{-jnt} dt = \frac{j}{2\pi n}, \] 
so that \( |D_n| = \frac{1}{2\pi n} \), and \( \angle D_n = \begin{cases} \frac{\pi}{2} & n > 0 \\ -\frac{\pi}{2} & n < 0 \end{cases} \)

3.5.2 In compact trigonometric form, all terms are of cosine form and amplitudes are positive. We can express \( f(t) \) as
\[ f(t) = 3 + 2 \cos \left( 2t - \frac{\pi}{3} \right) + \cos \left( 3t - \frac{\pi}{3} \right) + \frac{1}{2} \cos \left( 5t + \frac{\pi}{3} \right) \]
\[ = 3 + 2 \cos \left( 2t - \frac{\pi}{3} \right) + \cos \left( 3t - \frac{\pi}{3} \right) + \frac{1}{2} \cos \left( 5t - \frac{2\pi}{3} \right) \]

From this expression we sketch the trigonometric Fourier spectra as shown in Fig. 3.5.2a. By inspection of these spectra, we sketch the exponential Fourier spectra shown in Fig. 3.5.2b. From these exponential spectra, we can now write the exponential Fourier series as
\[ f(t) = 3 + e^{j(2t - \frac{\pi}{3})} + e^{j(3t - \frac{\pi}{3})} + \frac{1}{2} e^{j(5t - \frac{\pi}{3})} + \frac{1}{4} e^{j(2t - \frac{\pi}{3})} + \frac{1}{4} e^{-j(3t - \frac{\pi}{3})} \]

Fig. 3.5.2