1.4-6 (a) Recall that the derivative of a function at the jump discontinuity is equal to an impulse of strength equal to the amount of discontinuity. Hence, \( \frac{df}{dt} \) contains impulses \( 4\delta(t+4) \) and \( 2\delta(t-2) \). In addition, the derivative is \(-1\) over the interval \((-4, 0)\), and is \(1\) over the interval \((0, 2)\). The derivative is zero for \(t < -4\) and \(t > 2\). The result is sketched in Fig. S1.4-6a.

(b) Using the procedure in part (a), we find \( \frac{d^2f}{dt^2} \) for the signal in Fig. P1.4-2a as shown in Fig. S1.4-6b.

Fig. S1.4-6

1.4-7 (a) Recall that the area under an impulse of strength \(k\) is \(k\). Over the interval \(0 \leq t \leq 1\), we have

\[ y(t) = \int_0^t 1\, dx = t \quad 0 \leq t \leq 1 \]

Over the interval \(0 \leq t < 3\), we have

\[ y(t) = \int_0^t 1\, dx + \int_1^t (-1)\, dx = 2 - t \quad 1 \leq t < 3 \]

At \(t = 3\), the impulse (of strength unity) yields an additional term of unity. Thus,

\[ y(t) = \int_0^1 1\, dx + \int_1^3 (-1)\, dx + \int_3^t \delta(x-3)\, dx = 1 + (-2) + 1 = 0 \quad t > 3 \]

(b)

\[ y(t) = \int_0^t [1 - \delta(x-1) - \delta(x-2) - \delta(x-3) + \cdots]\, dx = tu(t) - u(t-1) - u(t-2) - u(t-3) - \cdots \]

Fig. S1.4-7
1.7-1: c) Is non-linear (0 input gives non-zero output). e) is nonlinear h) is linear

1.7-2: d) is time varying f) is time invariant

(d) The system is time-varying. The input \( f(t) \) yields the output \( y(t) = tf(t) \). For the input \( f(t - T) \), the output is \( tf(t - T) \), which is not \( tf(t) \) delayed by \( T \). Hence the system is time-varying.

(f) The system is time-invariant. The input \( f(t) \) yields the output \( y(t) \), which is the square of the second derivative of \( f(t) \). If the input is delayed by \( T \), the output is also delayed by \( T \). Hence the system is time-invariant.

2.4-11 (a) \( y(t) = e^{-t}u(t) + e^{-2t}u(t) = (e^{-t} - e^{-2t})u(t) \)

(c) \( e^{-2t}u(t-3) = e^{-6}e^{-2(t-3)}u(t-3) \). Now from the result in part (a) and the shift property of the convolution [Eq. (2.34)]:

\( y(t) = e^{-6}[e^{-3(t-3)}u(t) - e^{-2t-3}]u(t-3) \)

(d) \( f(t) = u(t) = u(t-1) \). Now \( y_1(t) \), the system response to \( u(t) \) is given by

\( y_1(t) = e^{-t}u(t) + u(t) = (1 - e^{-t})u(t) \)

The system response to \( u(t-1) \) is \( y_1(t-1) \) because of time-invariance property. Therefore the response \( y(t) \) to \( f(t) = u(t) - u(t-1) \) is given by

\( y(t) = y_1(t) - y_1(t-1) = (1 - e^{-t})u(t) - (1 - e^{-(t-1)})u(t-1) \)

The response is shown in Fig. S2.4-11.
(e) \[ c(t) = \int_{-\infty}^{1+t} \frac{1}{\pi^2 + 1} \, d\tau = \tan^{-1}(t-1) + \frac{\pi}{2}, \quad t \leq 1 \]
\[ c(t) = \int_{-\infty}^{0} \frac{1}{\pi^2 + 1} \, d\tau = \tan^{-1} t \bigg|_{-\infty}^{0} = \frac{\pi}{2}, \quad t \geq 1 \]

(h) \( f_1(t) = e^t, \quad f_2(t) = e^{-2t}, \quad f_1(\tau) = e^\tau, \quad f_2(t-\tau) = e^{-2(t-\tau)} \)

\[ c(t) = \int_{-1+t}^{0} e^\tau e^{-2(t-\tau)} \, d\tau = e^{-2t} \int_{-1+t}^{0} e^{3\tau} \, d\tau = \frac{1}{3} [e^{-2t} - e^{3t-3}], \quad 0 \leq t \leq 1 \]
\[ c(t) = \int_{-1+t}^{t} e^\tau e^{-2(t-\tau)} \, d\tau = e^{-2t} \int_{-1+t}^{t} e^{3\tau} \, d\tau = \frac{1}{3} [e^t - e^{3t-3}], \quad 0 \geq t \geq -1 \]
\[ c(t) = \int_{-2}^{t} e^\tau e^{-2(t-\tau)} \, d\tau = e^{-2t} \int_{-2}^{t} e^{3\tau} \, d\tau = \frac{1}{3} [e^t - e^{-2(t+3)}], \quad -1 \geq t \geq -2 \]
\[ c(t) = 0, \quad t \leq -2 \]